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NOUVELLE SÉRIE

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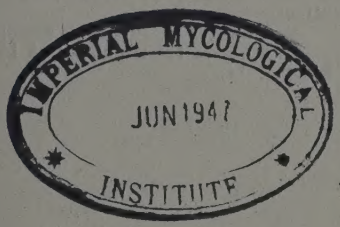

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MATHÉMATIQUES

SUR LE PRINCIPE DU MAXIMUM POUR LES SOLUTIONS D'UN SYSTÈME D'ÉQUATIONS A DÉRIVÉES PARTIELLES DU TYPE ELLIPTIQUE

Par G. ADELSON-WELSKY et A. KRONRODE

(Présenté par N. N. Lusin, de l'Académie, le 16. V. 1945)

Nous étudions dans cette Note les couples de fonctions réelles $u(x, y)$ et $v(x, y)$ vérifiant le système d'équations

$$\left.\begin{aligned} \frac{\partial u(x, y)}{\partial x} &= A(x, y)\,\frac{\partial v(x, y)}{\partial y} \\ \frac{\partial u(x, y)}{\partial y} &= B(x, y)\,\frac{\partial v(x, y)}{\partial x} \end{aligned}\right\} \tag{1}$$

telles que

$$A(x, y)\,A(\tilde{x}, \tilde{y}) > 0, \quad A(x, y)\,B(\tilde{x}, \tilde{y}) < 0 \tag{2}$$

où (x, y) et $(\tilde{x}, \tilde{y})$ sont les coordonnées de points arbitraires d'un domaine G, borné par une courbe C homéomorphe à une circonférence. D'ailleurs, les coefficients $A(x, y)$ et $B(x, y)$ sont arbitraires.

Soit $u(x, y)$ une fonction réelle définie dans un domaine fermé $\bar{G}$ et possédant une différentielle totale en chaque point de ce domaine.

Soient M et m le maximum et le minimum de $u(x, y)$ aux points de $\bar{G}$. En vertu du théorème 4 de notre Note[1] il existe sur le segment m, M un ensemble $\mathfrak{U}$ de mesure $M - m$ tel que pour $u \in \mathfrak{U}$ les composantes de l'ensemble de niveau E_u sont ou bien homéomorphes à une circonférence, ou bien présentent des arcs simples à extrémités sur C, ou bien des points. D'ailleurs, l'ensemble des points qui sont des composantes, et des points où E_u n'a pas de tangente, ont pour projections sur les axes x et y des ensembles de mesure nulle.

Lemme 1. Soit K_u une composante de E_u qui n'est pas un point $(u \in \mathfrak{U})$. Prenons sur K_u un paramètre t. En chaque point $a \in K_u$, où la tangente l_a à K_u existe, choisissons une direction n_a de telle manière que l'angle entre l_a et n_a, compté dans le sens positif, soit égal à $\pi/2$. Alors $\partial u / \partial n_a$ ne change pas de signe.

Démonstration. K_u divise G en deux domaines G_1 et G_2. Supposons qu'aux points a et b on a des tangentes à K_u. En vertu de ce que le plan peut être orienté, les points des rayons n_a et n_b, proches à a et b, se trouvent dans un même domaine, par exemple, dans G_1. Soit $\partial u / \partial n_a > 0$ et $\partial u / \partial n_b < 0$. Il existe un $\varepsilon > 0$ tel que pour $\xi \subset n_a$, $|\xi - a| < \varepsilon$ et $\eta \subset n_b$, $|\eta - b| < \varepsilon$ on a $u(\xi) > u(a) = u(b) > u(\eta)$. Alors les segments ξa et ηb sont séparés par une certaine composante $K'_u \subset E'_u$, et par conséquent $K_u \cap K'_u \neq 0$, donc $K_u = K'_u$, ce qui est impossible parce que K_u ne sépare pas les points ξ et η.

Théorème 1. *Soient $u(x, y)$ et $v(x, y)$ deux fonctions possédant des différentielles totales dans le domaine G. Soit M_{xy} l'ensemble des points du plan*

XY où l'on a

$$\frac{\partial u}{\partial x}=\frac{\partial u}{\partial y}=\frac{\partial v}{\partial x}=\frac{\partial v}{\partial y}=0$$

Soit M_{uv} l'ensemble de couples de valeurs des fonctions u et v aux points de l'ensemble M_{xy}. L'ensemble M_{uv} est mesurable dans le plan uv, et sa mesure plane est nulle.

Démonstration. Soit μ la mesure extérieure de M_{uv} et $\mu>0$. Soit S l'aire de G. Pour chaque point $a\in M_{xy}$ choisissons r_a de telle manière que pour $|b-a|<r_a$ on ait

$$\left|\frac{u(b)-u(a)}{b-a}\right|<T=\frac{1}{8}\sqrt{\frac{\mu}{S}} \tag{3}$$

Soit M_{uv}^r l'ensemble des points u_0, v_0 tels que pour chaque point $\xi(u, v_0)\in M_{uv}^r$ il existe un point $\eta(x, y)\in M_{xy}$, pour lequel $r_\eta>r$ et $u(x, y)=u_0$, $v(x, y)=v_0$.

Comme $M_{uv}=\sum_{n=1}^{\infty} M_{uv}^{r/n}$, il existe un r tel que la mesure extérieure de M_{uv}^r surpasse $\mu/2$. Soit M_{xy}^r l'ensemble des points $\eta(x, y)$ correspondants. Enfermons chaque point a de M_{uv}^r en un carré de centre a et dont le côté est parallèle à l'axe u et égal à $2rT$. Parmi ces carrés il existe au moins $\mu/2:9\cdot 4r^2T^2=\mu/72r^2T^2$ qui n'empiètent pas (en vertu du théorème de Vitali (2)). Entourons les points de M_{xy}^r correspondant aux centres des carrés choisis dans le plan uv, par des cercles de rayon r. L'aire de tous ces carrés n'empiétant pas est au moins égale à

$$\frac{\mu}{72r^2T^2}\pi r^2=\frac{64\pi\mu S}{72\mu}>S$$

ce qui démontre le théorème.

Définition 1. Si $F(x, y)$ est une fonction continue, l'ensemble G_c^+ est l'ensemble ouvert des points x_0, y_0, où $F(x_0, y_0)>C$, et G_c^- — l'ensemble ouvert des points x_1, y_1 tels que $F(x_1, y_1)<C$.

Lemme 2. Soient $u(x, y)$ et $v(x, y)$ deux fonctions possédant une différentielle totale et vérifiant les équations (1) dans le domaine G. Soient ensuite $\mathfrak{U}$ et $\mathfrak{V}$ les ensembles des valeurs de u et v pour lesquelles les ensembles de niveau E_u et $\tilde{E}_v$ vérifient les conditions, indiquées au commencement de cette Note. Soit K_u la composante de $E_u(u\in\mathfrak{U})$, et $\tilde{E}_v(v\in\mathfrak{V})$ — l'ensemble de niveau. Alors, si $K_u\cap\tilde{E}_v$ contient plus d'un point, il existe un point de $K_u\cap\tilde{E}_v$, où

$$\frac{\partial u}{\partial x}=\frac{\partial u}{\partial y}=\frac{\partial v}{\partial x}=\frac{\partial v}{\partial y}=0. \tag{4}$$

Démonstration. Introduisons sur K_u un paramètre t. Soit a le premier point d'intersection de K_u et $\tilde{E}_v$. Si (4) n'a pas lieu au point a, les ensembles K_u et $\tilde{E}_v$ possèdent au point a des tangentes l_a^u et l_a^v. Soit φ_a l'angle entre les gradients de u et v au point a. En tenant compte de (1), nous aurons

$$\sin\varphi_a=\frac{\frac{\partial v}{\partial y}\frac{\partial u}{\partial x}-\frac{\partial v}{\partial x}\frac{\partial u}{\partial y}}{|\operatorname{grad} u_a|\,|\operatorname{grad} v_a|}=\frac{A\left(\frac{\partial v}{\partial y}\right)^2-B\left(\frac{\partial v}{\partial x}\right)^2}{|\operatorname{grad} u_a|\,|\operatorname{grad} v_a|} \tag{5}$$

En vertu de (2) l'expression (5) conserve son signe quand le point a passe par les points d'intersection de K_u et $\tilde{E}_v$, où (4) n'a pas lieu. Soit b le premier point de $K_u\cap\tilde{E}_v$ qui suit a. L'ensemble de niveau $\tilde{E}_v$ divise G en

deux ensembles G_v^+ et G_v^-. Comme $\sin\varphi_a \neq 0$, la composante K_u au point a passe, par exemple, de G_v^- en G_v^+ (si le paramètre t croît). Si au point b (4) n'a pas lieu, on a $\sin\varphi_b \neq 0$, et K_u passe de G_v^+ en G_v^- au point b. Mais dans ce cas $\sin\varphi_a \sin\varphi_b < 0$. Cette contradiction démontre le théorème.

Théorème 2 (sur le principe du maximum). *Soient $u(x, y)$ et $v(x, y)$ deux fonctions possédant une différentielle totale et vérifiant les équations* (1) *dans le domaine G. Alors $u(x, y)$ atteint sa valeur maximale sur la frontière C du domaine G.*

Démonstration. Soit $M = u(a) > \max\limits_{\xi \subset C} u(\xi) = m$. Alors tous les ensembles E_u pour $u \in \mathfrak{U}$, où $\mathfrak{U}$ est situé sur $m\overline{M}$ et de mesure $M - m$, contiennent seulement des composantes homéomorphes à une circonférence ou bien formées d'un seul point. Considérons l'ensemble M_{uv} de mesure null, défini dans le théorème 1. Choisissons $u_0 \in \mathfrak{U}$ de telle manière que l'ensemble $M_{uv} \cap l_{u_0}$, où l_{u_0} est la droite $u = u_0$ dans le plan uv, soit de mesure nulle. Alors une composante $K_{u_0} \subset E_{u_0}$, qui n'est pas un point, coupe chaque ensemble de niveau $\tilde{E}_v$ pour $n < v < N$, où n et $N > n$ sont le minimum et le maximum de $v(\xi)$ $(\xi \subset K_{u_0})$. Comme K_{u_0} est homéomorphe à une circonférence, son intersection avec chaque $\tilde{E}_v$ (sauf un ensemble de niveaux v de mesure nulle) contient au moins deux points. Le lemme 2 conduit à une contradiction, donc le théorème est démontré.

Remarque 1. Le théorème 2 reste vrai, si l'on remplace dans son énoncé le mot maximal par minimal.

Remarque 2. Le théorème 2 reste vrai, si l'on remplace les équations (1) par les équations

$$\left.\begin{aligned} \frac{\partial u}{\partial x} &= a_{11}(x, y)\frac{\partial v}{\partial x} + a_{12}(x, y)\frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} &= a_{21}(x, y)\frac{\partial v}{\partial x} + a_{22}(x, y)\frac{\partial v}{\partial y} \end{aligned}\right\} \qquad (1')$$

en supposant d'ailleurs

$$\left.\begin{aligned} a_{12}(x, y)\, a_{12}(\tilde{x}, \tilde{y}) &> 0 \\ 4a_{12}a_{21} + (a_{22} - a_{11})^2 &< 0 \end{aligned}\right\} \qquad (2')$$

où (x, y) et $(\tilde{x}, \tilde{y})$ sont des points arbitraires du domaine G. Les démonstrations restent dans ce cas les mêmes.

Remarque 3. Les conditions du théorème 2 sont symétriques par rapport aux fonctions $u(x, y)$ et $v(x, y)$, donc le principe du maximum est encore vrai pour la fonction $v(x, y)$.

Remarque 4. Dans le théorème 2 on ne suppose pas que les coefficients sont continus et, d'ailleurs, on peut supposer qu'ils dépendent de u, v, $\partial u/\partial x$, $\partial u/\partial y$, $\partial v/\partial x$, $\partial v/\partial y$, ...

Université Lomonosov.
Moscou.

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LITTÉRATURE CITÉE

[1] G. M. Adelson-Welsky et A. S. Kronrode, C. R. Acad. Sci. URSS, XLIX, No. 4 (1945). [2] Ch. J. de la Vallée-Poussin, Cours d'analyse infinitésimale, 2, 1933.

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MATHEMATICS

ON A THEOREM OF N. WIENER

By S. LOZINSKI

(Communicated by V. I. Smirnov, Member of the Academy, 10. III. 1945)

1. Let V be the set of all real and 2π-periodic functions which are of bounded variation on the interval $[0, 2\pi]$. Let $\overline{V}$ denote the set of those functions $f \in V$ which satisfy at every point x the relation

$$\min [f(x-0),\ f(x+0)] \leqslant f(x) \leqslant \max [f(x-0),\ f(x+0)] \tag{1}$$

If

$$f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx) \tag{2}$$

is the Fourier series of the function $f(x)$, then we put

$$\rho_n = \sqrt{a_n^2 + b_n^2}, \qquad n = 1, 2, \ldots \tag{3}$$

2. The following facts are known:

1) If $f \in V$, then

$$\rho_n = O\left(\frac{1}{n}\right) \tag{4}$$

2) If $f(x)$ is 2π-periodic and absolutely continuous on the interval $[0, 2\pi]$, then

$$\rho_n = o\left(\frac{1}{n}\right) \tag{5}$$

3) If $f \in V$ and is continuous (but not absolutely continuous), then (5) does not necessarily hold (1).

N. Wiener has proved (2) the following theorem: if $f \in \overline{V}$, then in order that f be continuous it is necessary and sufficient that

$$\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} k\rho_k = 0 \tag{6}$$

3. In view of 1) and 2) it is obvious that

1') If $f \in V$, then

$$\sum_{k=1}^{n} \rho_k = O(\log n) \tag{7}$$

2') If f is 2π-periodic and absolutely continuous on $[0, 2\pi]$, then

$$\sum_{k=1}^{n} \rho_k = o(\log n) \tag{8}$$

It is the purpose of the present note to prove the following

Theorem. *If $f \in \overline{V}$, then in order that f be continuous it is necessary and sufficient that* (8) *should hold.*

The proof is based on the following lemmas.

Lemma 1. Let $\nu_1, \nu_2, \ldots$ be a sequence of positive integers, $1 \leqslant \nu_1 < \nu_2 < \ldots$ and let $N(n)$ denote the number of those integers $\nu_1, \nu_2, \ldots$ which are $\leqslant n$. Then

1) If

$$\lim_{n\to\infty} \frac{N(n)}{n} = 0$$

then

$$\lim_{n\to\infty} \frac{\frac{1}{\nu_1} + \frac{1}{\nu_2} + \ldots + \frac{1}{\nu_{N(n)}}}{1 + \frac{1}{2} + \ldots + \frac{1}{n}} = 0$$

2) If

$$\lim_{n\to\infty} \frac{N(n)}{n} > 0$$

then

$$\lim_{n\to\infty} \frac{\frac{1}{\nu_1} + \frac{1}{\nu_2} + \ldots + \frac{1}{\nu_{N(n)}}}{1 + \frac{1}{2} + \ldots + \frac{1}{n}} > 0$$

The very simple proof of this lemma is left to the reader.

The following lemma is a partial generalization of the theorem of N. Wiener.

Lemma 2. If $f \in \overline{V}$, then in order that f be continuous it is (necessary and) sufficient that

$$\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} k\rho_k = 0 \tag{9}$$

The proof is similar to the proof of the sufficiency of the relation (6) as given by Zygmund [(3), § 9, 7]. We put, with Zygmund,

$$B_n = \frac{1}{n} \sum_{k=1}^{n} k^2 \rho_k^2, \quad C_n = n \sum_{k=1}^{\infty} \rho_k^2 \sin^2 \frac{k\pi}{2n}$$

The proof of Zygmund implies that

1) If $f \in \overline{V}$ and

$$\lim_{n\to\infty} C_n = 0 \tag{10}$$

then f is continuous.

2) If $f \in V$ and (9) holds, then

$$\lim_{n\to\infty} B_n = 0 \tag{11}$$

Hence we have only to prove that $f \in \overline{V}$ and (11) imply (10).

It can evidently be assumed that $\operatorname{var} f \leqslant 1$, where $\operatorname{var} f$ denotes the total variation of $f(x)$ on $[0, 2\pi]$. There exists by supposition an increasing sequence of positive integers $\{n_i\}_{i=1}^{\infty}$ such that $B_{n_i} \to 0$ for $i \to \infty$. Clearly

$$C_n \leqslant n \sum_{k=1}^{n_i} \rho_k^2 \left(\frac{k\pi}{2n}\right)^2 + n \sum_{k=n_i+1}^{\infty} \frac{1}{k^2} =$$

$$= \frac{1}{n} \frac{\pi^2}{4} \sum_{k=1}^{n_i} k^2 \rho_k^2 + \frac{n}{n_i} = \frac{n_i}{n} \frac{\pi^2}{4} B_{n_i} + \frac{n}{n_i} \tag{12}$$

Let $\varepsilon > 0$ be given. For $i = 1, 2, \ldots$ we put $m_i = [\varepsilon n_i]$, where $[\varepsilon n_i]$ means the greatest integer not exceeding εn_i.

Taking in (12) $n = m_i$, we obtain

$$C_{m_i} \leqslant \frac{n_i}{[\varepsilon n_i]} \frac{\pi^2}{4} B_{n_i} + \varepsilon$$

It follows that $\overline{\lim_{i\to\infty}} C_{m_i} \leqslant \varepsilon$, *i. e.* $\overline{\lim_{n\to\infty}} C_n \leqslant \varepsilon$. Since $\varepsilon > 0$ is arbitrary small, we have (10), *q. e. d.*

Lemma 3. Let $f \in V$. If $\varepsilon > 0$ is fixed, we denote by $N_\varepsilon(n)$ the number of those indices $\nu = 1, 2, \ldots$ for which $\nu\rho_\nu \geqslant \varepsilon$.

Then, for $n = 1, 2, \ldots$ and every $\varepsilon > 0$ we have

$$\varepsilon \frac{N_\varepsilon(n)}{n} \leqslant \frac{1}{n} \sum_{\nu=1}^{n} \nu\rho_\nu \leqslant \varepsilon + \frac{N_\varepsilon(n)}{n} \operatorname{var} f \tag{13}$$

Denoting, further, by $\nu_1, \nu_2, \ldots$ those indices ν for which $\nu\rho_\nu \geqslant \varepsilon$, we have

$$\varepsilon \left(\frac{1}{\nu_1} + \frac{1}{\nu_2} + \ldots + \frac{1}{\nu_{N_\varepsilon(n)}}\right) \leqslant \sum_{k=1}^{n} \rho_k \leqslant \varepsilon \sum_{k=1}^{n} \frac{1}{k} + \left(\frac{1}{\nu_1} + \frac{1}{\nu_2} + \ldots + \frac{1}{\nu_{N_\varepsilon(n)}}\right) \operatorname{var} f \tag{14}$$

The proof is left to the reader.

The sequence $\nu_1, \nu_2, \ldots$ depends evidently on ε so that we should, strictly speaking, write $\nu_1^{(\varepsilon)}, \nu_2^{(\varepsilon)}, \ldots$ But it is hoped that no misunderstanding shall arise from our notation.

Now we can prove our theorem. If $f \in \overline{V}$ and (8) holds, then by (14) for every fixed $\varepsilon > 0$ we have

$$\lim_{n\to\infty} \frac{\frac{1}{\nu_1} + \frac{1}{\nu_2} + \ldots + \frac{1}{\nu_{N_\varepsilon(n)}}}{1 + \frac{1}{2} + \ldots + \frac{1}{n}} = 0 \tag{15}$$

By Lemma 1 we have for every $\varepsilon = 0$

$$\lim_{n\to\infty} \frac{\nu_{(n)}}{n} = 0 \tag{16}$$

By (13) we have (9) and, hence, by Lemma 2 the function f is continuous. The sufficiency of (8) is thus proved. Let us now suppose that $f \in V$ and is continuous. By the theorem of N. Wiener and by (13) we have for every $\varepsilon > 0$

$$\lim_{n\to\infty} \frac{N_\varepsilon(n)}{n} = 0$$

By Lemma 1 we have (15) for every $\varepsilon > 0$. Since $\varepsilon > 0$ is arbitrarily small, (14) implies (8), which proves the necessity of condition (8).

Remark 1. Our theorem admits of the same partial generalization as the theorem of N. Wiener: if $f \in \overline{V}$, then in order that f be continuous it is necessary and sufficient that

$$\lim_{n\to\infty} \frac{1}{\log n} \sum_{k=1}^{n} \rho_k = 0$$

The proof of this is contained in the proof of our theorem.

Remark 2. The conditions (6) and (8) are quite similar. It might seem that they are equivalent in the following sense: if $\{\rho_\nu\}_{\nu=1}^{\infty}$ is a sequence of

non-negative numbers that satisfy the condition (4), then in order that (6) should hold, it is necessary and sufficient that (8) hold. But this is not true. Although — as can be easily seen from the proof of our theorem — (4) and (6) imply (8), but (4) and (8) do not imply (6).

Example.

$$\rho_\nu = \begin{cases} \frac{1}{\nu} & \text{for} \quad 2^{n^2} < \nu \leqslant 2^{n^2+1} \\ 0 & \text{for every other } \nu \end{cases} \qquad (n = 1, 2, \ldots)$$

Then, if $N > 2$ is an integer, we take the greatest integer n such that $2^{n^2} < N$.

We have

$$\frac{1}{\log N} \sum_{\nu=1}^{N} \rho_\nu \leqslant \frac{1}{n^2 \log 2} \sum_{\nu=1}^{2^{n^2+1}} \rho_\nu \leqslant \frac{1}{n^2 \log 2} \sum_{k=1}^{n} \sum_{\nu=1}^{2^{k^2}} \frac{1}{2^{k^2}+\nu} \leqslant \frac{1}{n^2 \log 2}\, n \log 2 = \frac{1}{n} \longrightarrow 0$$

for $N \longrightarrow \infty$, *i. e.* the sequence $\{\rho_\nu\}_{\nu=1}^{\infty}$ satisfies (4) and (8). But this sequence does not satisfy the condition (6), for

$$\frac{1}{2^{n^2+1}} \sum_{\nu=1}^{2^{n^2+1}} \nu\rho_\nu \geqslant \frac{1}{2^{n^2+1}} \sum_{\nu=2^{n^2}+1}^{2^{n^2+1}} \nu\rho_\nu = \frac{2^{n^2}}{2^{n^2+1}} = \frac{1}{2}$$

i. e. the conditions (6) and (8) are not equivalent even if (4) holds. Our theorem shows that they become equivalent if the sequence or numbers $\{\rho_\nu\}_{\nu=1}^{\infty}$ possesses a transcendental property — is the sequence of absolute values of the complex Fourier coefficients of a function $f \in V$.

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REFERENCES

[1] F. Riesz, Mathem. Z., 2, 312 (1918). [2] N. Wiener, Massachusetts J. of Mathem., 3, 72 (1924). [3] A. Zygmund, Trigonometrical series, Monografie Matematyszne, V, 1935.

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MATHÉMATIQUES

LA LOI DIRECTE, INVERSE ET ABSOLUE DES GRANDS NOMBRES

Par A. LOURIER

(Présenté par A. N. Kolmogoroff, de l'Académie, le 16. VII. 1945)

§ 1. Soit $\Phi(y \mid n, \zeta)$ une suite de lois de distribution des variables aléatoires $\eta_n (n = 1, 2, 3, \ldots, \infty)$, définie par un paramètre ζ. Soit ξ_n une fonctionnelle $\mathfrak{F}(\Phi)$ de $\Phi(y \mid n, \zeta)$. Donc, $\xi_n = f(n, \zeta)$. Désignons par $\theta(z)$ la loi de distribution de la variable aléatoire ζ. Alors dans certaines hypothèses naturelles concernant la «mesurabilité» de Φ et $\mathfrak{F}$, il existe une suite de lois absolues de distribution des variables aléatoires η_n

$$\Phi(y \mid n) = \int_{-\infty}^{\infty} \Phi(y \mid n, \zeta = z)\, d\theta(z)$$

et une suite de lois absolues de distribution $F(x \mid n)$ des variables aléatoires ξ_n.

On appelle événement A_n la réalisation de l'inégalité $|\eta_n - \xi_n| < \varepsilon$, $\varepsilon > 0$. Désignons respectivement par $P(A_n \mid z)$, $P(A_n \mid x_n)$, $P(A_n \mid y_n)$ et $P(A_n)$ les probabilités

$P(|\eta_n - \xi_n| < \varepsilon \mid \zeta = z)$ où z est un nombre réel
$P(|\eta_n - \xi_n| < \varepsilon \mid \xi_n = x_n)$ où x_n est une suite de nombres réels
$P(|\eta_n - \xi_n| < \varepsilon \mid \eta_n = y_n)$ où y_n est une suite de nombres réels

et la probabilité absolue $P(|\eta_n - \xi_n| < \varepsilon)$.

Il est facile de voir que dans les hypothèses faites ces probabilités existent.

On appelle loi directe des grands nombres les théorèmes qui affirment que si une famille de suites des fonctions $\Phi(y \mid n, \zeta)$ vérifie certaines conditions, alors pour chaque valeur z du paramètre ζ on a

$$P(A_n \mid z) \underset{n \to \infty}{\longrightarrow} 1 \tag{1}$$

Nous appellerons loi inverse des grands nombres les théorèmes qui affirment que dans certaines conditions

$$P(A_n \mid y_n) \underset{n \to \infty}{\longrightarrow} 1 \tag{2}$$

Dans les théorèmes directes c'est η_n qui est la variable aléatoire, dans les théorèmes inverses c'est ξ_n, c'est-à-dire on considère comme événement aléatoire le fait que η_n vérifie une loi de distribution possédant ξ_n pour valeur de la fonctionnelle. En considérant la probabilité de l'événement A_n dans l'hypothèse que η_n vérifie une certaine loi de distribution et que la valeur

de τ_{in} est un événement aléatoire, on obtient une expression limite qui sera nommée loi absolue des grands nombres

$$P(A_n) \underset{n\to\infty}{\longrightarrow} 1 \tag{3}$$

Les théorèmes concernant la loi absolue des grands nombres présentent un certain intérêt pour le fondement des principes des méthodes statistiques (comparer les remarques de A. Kolmogoroff [1]) sur le rôle des probabilités absolues dans les fondements rigoureux de l'application des probabilités «fiduciaires» de Fischer).

§ 2. Théorème 1. *Si l'on a pour chaque valeur z du paramètre* ζ

$$P(A_n \mid z) \underset{n\to\infty}{\longrightarrow} 1$$

alors

$$P(A_n) \underset{n\to\infty}{\longrightarrow} 1$$

Démonstration. Soit m un entier positif. Désignons par z_m toutes les valeurs du paramètre ζ pour lesquelles, quel que soit le nombre positif ε, on a les inégalités

$$\begin{aligned} &P(A_n \mid z_m) > 1-\varepsilon, \quad n \geqslant m \\ &P(A_n \mid z_m) \leqslant 1-\varepsilon, \quad n = m-1 \end{aligned} \tag{C}$$

Il est facile de voir que

$$P(A_n) > (1-\varepsilon) \sum_{m=1}^{m=n} P(z_m) \tag{4}$$

où $P(z_m)$ sont les probabilités des valeurs correspondantes de ζ, définies par la condition (C).

L'existence des probabilités $P(z_m)$ résulte des hypothèses faites dans le § 1. En vertu de (1) chaque z appartient à l'un des ensembles z_m pour $m = 1, 2, 3, \ldots, \infty$. Donc, $\sum_{m=1}^{m=n} P(z_m) \underset{n\to\infty}{\longrightarrow} 1$. Il en résulte immédiatement que le théorème est démontré *.

Remarque. Si l'on suppose que la loi de distribution du paramètre ζ dépend de n, donc si les $P(z_m)$ dépendent de n, alors pour que (3) ait lieu il faut que l'on ait

$$\sum_{m=1}^{m=n} P(z_m \mid n) \underset{n\to\infty}{\longrightarrow} 1$$

En particulier, le théorème 1 subsiste si $P(A_n \mid z) \to 1$ uniformément pour $n \to \infty$.

Théorème 2. *Si*

$$P(A_n) \underset{n\to\infty}{\longrightarrow} 1$$

---

* Le théorème 1 peut aussi être démontré de la manière suivante: $P(A_n) = \int_{-\infty}^{\infty} P(A_n \mid z) d\theta(z)$.

L'existence de cette intégrale est une conséquence des hypothèses du § 1. Il est facile de voir que $P(A_n \mid z) \to 1$ pour $n \to \infty$ entraîne

$$\int_{-\infty}^{\infty} P(A_n \mid z) d\theta(z) \underset{n\to\infty}{\longrightarrow} \int_{-\infty}^{\infty} d\theta(z) = 1$$

on a

$$P\{P(\overline{A}_n \mid y_n) > \varepsilon_0\} \underset{n\to\infty}{\longrightarrow} 0, \qquad \varepsilon_0 > 0 \tag{5}$$

où $\overline{A}_n$ *est l'événement contraire à* A_n, *c'est-à-dire la réalisation de l'inégalité* $|\eta_n - \xi_n| \geqslant \varepsilon$.

En d'autres termes, le théorème 2 affirme que la probabilité des valeurs de η_n, pour lesquelles la probabilité $\overline{A}_n$ surpasse un nombre positif arbitraire ε_0, donc la probabilité de ce que la probabilité A_n soit inférieure à $1-\varepsilon_0$, tend vers zéro pour $n \to \infty$.

Démonstration. $P(\overline{A}_n)$ est la probabilité absolue de l'événement $\overline{A}_n$, c'est donc l'espérance mathématique de la probabilité $P(\overline{A}_n \mid y_n)$. Il résulte de (3) que $P(\overline{A}_n) \to 0$ pour $n \to \infty$. Or, si l'espérance mathématique d'une variable aléatoire non négative tend vers zéro, la probabilité des valeurs de cette variable surpassant un nombre positif arbitraire tend aussi vers zéro.

Le théorème 2 présente une généralisation d'une certaine remarque de Bernstein [2].

On démontre d'une manière analogue le

Théorème 3. *Si*

$$P(A_n) \underset{n\to\infty}{\longrightarrow} 1$$

on a

$$P\{P(\overline{A}_n \mid z) > \varepsilon_0\} \underset{n\to\infty}{\longrightarrow} 0, \qquad \varepsilon_0 > 0 \tag{6}$$

§ 3. Dans les théorèmes sur la loi inverse des grands nombres on suppose connues des valeurs déterminées de la variable aléatoire η_n ($\eta_n = y_n$). Modifions cette hypothèse. Nous allons supposer que l'on ne connaît pas les valeurs déterminées de η_n, mais certains intervalles aussi petits que l'on veut $(y_n - \varepsilon_0,\ y_n + \varepsilon_0)$, $\varepsilon_0 > 0$, qui contiennent les valeurs possibles de η_n. On peut démontrer le théorème suivant très général sur «la loi universe modifiée des grands nombres».

Théorème 4. *Si*

$$P(A_n) \underset{n\to\infty}{\longrightarrow} 1$$

on a

$$P(|\eta_n - \xi_n| < \varepsilon \mid y_n - \varepsilon_0 < \eta_n < y_n + \varepsilon_0) \underset{n\to\infty}{\longrightarrow} 1, \qquad 0 < \varepsilon_0 < \varepsilon \tag{7}$$

ou, en abrégé,

$$P(A_n \mid y_n,\ \varepsilon_0) \underset{n\to\infty}{\longrightarrow} 1 \tag{7a}$$

pourvu que

$$F(y_n + h \mid n) - F(y_n - h \mid n) > \lambda_h, \qquad h > 0,\ \lambda_h > 0 \tag{D}$$

Démonstration. En vertu du théorème de Bayes, on a

$$P(A_n \mid y_n,\ \varepsilon_0) = \frac{\int_{y_n-\varepsilon}^{y_n+\varepsilon} P(|\eta_n - y_n| < \varepsilon_0 \mid \xi_n = x)\, dF(x \mid n)}{\int_{-\infty}^{\infty} P(|\eta_n - y_n| < \varepsilon_0 \mid \xi_n = x)\, dF(x \mid n)} \tag{8}$$

Donc, pour que (7) ait lieu, il faut que l'on ait

$$\frac{\int_{|y_n - x| \geqslant \varepsilon} P(|\eta_n - y_n| < \varepsilon_0 \mid \xi_n = x)\, dF(x \mid n)}{\int_{y_n-\varepsilon}^{y_n+\varepsilon} P(|\eta_n - y_n| < \varepsilon_0 \mid \xi_n = x)\, dF(x \mid n)} \underset{n\to\infty}{\longrightarrow} 0 \tag{9}$$

Soit $\varepsilon_1 = \varepsilon - \varepsilon_0$. En vertu du théorème 3, la probabilité des suites de lois de distribution η_n — des valeurs z du paramètre ζ, telles que $P(|\eta_n - \xi_n| \geqslant \varepsilon_1 | z) > \varepsilon'$ ($\varepsilon' > 0$), tend vers zéro pour $n \to \infty$. Désignons cette probabilité par α_n.

En vertu du même théorème 3 la probabilité des suites de lois de distribution η_n — des valeurs z du paramètre ζ, telles que $P\left(|\eta_n - \xi_n| < \frac{\varepsilon_0}{2} \middle| z\right) < 1 - \varepsilon'$, tend aussi vers zéro pour $n \to \infty$. Soit β_n cette probabilité.

En tenant compte de la condition (D), on peut démontrer l'inégalité suivante

$$\frac{\int\limits_{|y_n - x| \geqslant \varepsilon} P(|\eta_n - y_n| < \varepsilon_0 | \xi_n = x)\, dF(x|n)}{\int\limits_{y_n - \varepsilon}^{y_n + \varepsilon} P(|\eta_n - y_n| < \varepsilon_0 | \xi_n = x)\, dF(x|n)} < \frac{\varepsilon' \int\limits_{|y_n - x| \geqslant \varepsilon} dF(x|n) + \alpha_n}{(1 - \varepsilon') \int\limits_{x_n - \varepsilon_0/2}^{y_n + \varepsilon_0/2} dF(x|n) - \beta_n} < \frac{\varepsilon' + \alpha_n}{(1 - \varepsilon')\lambda_{\varepsilon_0/2} - \beta_n} \tag{10}$$

Comme ε' peut être choisi aussi petit que l'on veut, $\lambda_{\varepsilon_0/2}$ est constant, $\alpha_n \to 0$, $\beta_n \to 0$ pour $n \to \infty$, il résulte immédiatement de (10) que (9), donc aussi (7), est vrai.

Corollaire 1. *Si $P(A_n|z) \to 1$ pour $n \to \infty$ et pour toutes les valeurs de z, c'est-à-dire si la loi directe des grands nombres a lieu, alors la condition* (D) *est suffisante pour que la loi inverse modifiée des grands nombres ait lieu.*

Cela résulte de la comparaison des théorèmes 1 et 4.

Corollaire 2. *Supposons que la distribution de ξ_n ne dépend pas de n, c'est-à-dire que $F(x|n) = F(x)$. Alors, si $P(A_n) \to 1$ pour $n \to \infty$ et en particulier si pour toutes les valeurs de z on a $P(A_n|z) \to 1$ pour $n \to \infty$, pour que la relation «$P(A_n | y_n, \varepsilon_0) \to 1$ pour $n \to \infty$ a lieu uniformément pour $a \leqslant y_n \leqslant b$» soit vérifiée, il est nécessaire et suffisant que $F(x)$ soit une fonction croissante dans (a, b).*

En effet, dans ce cas pour tous les y_n situés dans (a, b) la condition (D) est vérifiée.

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LITTÉRATURE CITÉE

[1] A. N. Kolmogoroff, Bull. Acad. Sci. URSS, sér. mathém., **6**, No. 1—2, 3 (1942).
[2] S. N. Bernstein, ibid., **5**, No. 2, 85 (1941).

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FLUID DYNAMICS

A CONTRIBUTION TO THE THEORY OF THE IDEAL CAVITATING SCREW PROPELLER

By A. M. BASIN

(Communicated by V. L. Posdunine, Member of the Academy, 5. I. 1945)

The creation of the above theory has been suggested by V. L. Posdunine. In his paper (1) we also find a brief description of the physical phenomena involved in the work of cavitating propellers. His hydrodynamic model of the operation of such propellers was somewhat different from that which will be discussed presently, and accordingly the conclusions are not identical with his. The author is going to prove theoretically the possibility of conditions for a negative change of pressure in the work of cavitating screws. This statement was first advanced by Posdunine, and made the basis of his hydrodynamic model of the work of cavitating screw propellers.

1. Problem Stated. In order to reveal the general laws underlying the work of a cavitating propeller, we shall consider the flow created by it proceeding on the following assumptions: a) the flow has no viscosity, being formed by an ideal fluid; b) all velocities in the flow are supposed to be axial, directed opposite to the thrust, and any rotary motion is absent.

We assume further that the density of the flow created by the propeller ρ_1 is constant and not necessarily equal to the density of water ρ.

Here we take due account of the fact that in the case of cavitation the continuous flow of water disintegrates within the screw into several sheets which correspond quantitatively to the number of blades and flow into a space filled with a mixture of air and water vapour.

If there are many blades, the sheets are small in dimensions, and it may be assumed as an approximation that the flow cast by the screw is a homogeneous mixture of air and water whose average density ρ_1 is lower than the density of the water ρ. On account of the sudden change of density in the cross-section of the screw the velocity and pressure of the fluid in the screw should likewise change abruptly.

Let us consider the motion of the fluid in a system of coordinates fixed to the propeller. Denote the cross-section of the screw by $0-0$; the density, pressure and velocity of the fluid at an infinite distance afore the screw, by ρ, p_0, v_0, respectively; just in front of the screw, by ρ, p_1, v_s; just behind the screw, by ρ_1, p_2, $v_s+\Delta v$; far abaft the screw, by ρ_1, p_0, v_∞; the quantity Δv is the velocity increment in the screw.

What stands out as the most characteristic feature in the here considered hydrodynamical model of an ideal cavitating propeller, is the sudden change of pressure in the screw disc.

Let us determine the main characteristics of the work of our ideal cavitating propeller, *viz.*: a) the velocity of water inflow to the screw v_s; b) the

change of pressure in the disc of the screw $\Delta p = p_2 - p_1$; c) the velocity of the flow far abaft the screw v_∞; d) the thrust P; e) the efficiency of the screw η. All these values we shall express as functions of the thrust coefficient

$$\sigma_p = \frac{2P}{\rho F_p v_0^2} \tag{1}$$

2. Thrust. In determining the value of the instantaneous change of pressure in the screw disc, we can apply Bernoulli's equation to the flow afore and abaft the screw in succession

$$p_0 + \rho \frac{v_0^2}{2} = p_1 + \rho \frac{v_s^2}{2}, \quad p_0 + \rho_1 \frac{v_\infty^2}{2} = p_2 + \rho_1 \frac{(v_s + \Delta v)^2}{2}$$

Hence

$$p_2 - p_1 = \rho_1 \frac{v_\infty^2}{2} - \rho \frac{v_0^2}{2} - \rho_1 \frac{(v_s + \Delta v)^2}{2} + \rho \frac{v_s^2}{2} \tag{2}$$

The instantaneous increment of velocity is determined from the condition that the discharge before and behind the screw should be constant. We have

$$\rho v_s F_p = \rho_1 (v_s + \Delta v) F_p$$

Hence

$$\Delta v = v_s \left(\frac{\rho}{\rho_1} - 1 \right) \tag{3}$$

The thrust of the ideal cavitating propeller will be determined in two ways: a) by applying the law of momentum to a control surface extended around the propeller at an infinite distance (say, the surface of a sphere of very large radius); b) by applying the law of momentum to a control surface running close before and behind the propeller.

The first method will give us the following expression for the thrust

$$P = \rho_1 F_\infty v_\infty (v_\infty - v_0) \tag{4}$$

Here F_∞ is the sheet area at an infinite distance abaft the screw. Its value is determined from the condition that the discharge should be constant in the sheet. We have

$$\rho_1 F_\infty v_\infty = \rho_1 F_p (v_s + \Delta v) = \rho F_p v_s \tag{5}$$

Substituting (5) in (4) we find

$$P = \rho F_p v_s (v_\infty - v_0) \tag{6}$$

The expression for the thrust obtained by the second method is

$$P = (p_2 - p_1) F_p + \rho F_p v_s \Delta v \tag{7}$$

Substituting in (7) the values of $p_2 - p_1$ and Δv from (2) and (3) we find

$$P = \rho_1 F_p \frac{v_\infty^2}{2} - \rho F_p \frac{v_0^2}{2} - \rho F_p \frac{v_s^2}{2} \left(\frac{\rho}{\rho_1} - 1 \right) \tag{8}$$

Equating the two expressions obtained for the thrust, *viz.* (6) and (8), we find the relation between the velocity of water inflow to the screw v_s and the velocity of flow at an infinite distance abaft the screw v_∞. This relation is obtained in the form of a quadratic equation

$$\left(\frac{v_\infty}{v_0} \right)^2 - 2 \frac{\rho}{\rho_1} \frac{v_s}{v_0} \frac{v_\infty}{v_0} - \frac{\rho}{\rho_1} \left(\frac{v_s}{v_0} - 1 \right)^2 + \frac{\rho}{\rho_1} \left(\frac{v_0}{v_0} \right)^2 = 0$$

Its solution gives the required relation

$$\frac{v_\infty}{v_0} = \frac{\rho}{\rho_1} \frac{v_s}{v_0} + \left(\frac{v_s}{v_0} - 1 \right) \sqrt{\frac{\rho}{\rho_1}} \tag{9}$$

The radical is taken with the plus sign because $v_\infty > v_s$.

We will now express the pressure change in the screw as a function of ρ/ρ_1 and v_s/v_0. Substituting (9) and (3) in (2) we find

$$\frac{2\Delta p}{\rho v_0^2} = 2\frac{v_s}{v_0}\left(\frac{v_s}{v_0} - 1\right)\left(\sqrt{\frac{\rho}{\rho_1}} + 1\right) \tag{10}$$

It follows from formula (10) that the existence of conditions for a negative pressure change is a reality in a working cavitating propeller. These conditions are set up when $v_s/v_0 < 1$.

Let us find out the relation between the relative velocity v_s/v_0 and the thrust coefficient σ_p. For this we express the thrust as a function of ρ/ρ_1 and v_s/v_0. Substituting (9) in the expression for the thrust we find

$$P = \rho F_p v_s (v_s - v_0)\left(\sqrt{\frac{\rho}{\rho_0}} + 1\right) + \rho F_p v_s^2\left(\frac{\rho}{\rho_1} - 1\right) \tag{11}$$

Introducing in (11) the thrust coefficient σ_p, we obtain the relation between v_s/v_0 and σ_p in the form of a quadratic equation

$$\left(\frac{v_s}{v_0}\right)^2 - 2\left(\frac{v_s}{v_0}\right)\frac{1}{2\sqrt{\frac{\rho_1}{\rho}}} - \sigma_p \frac{1}{2\left(\frac{\rho}{\rho_1} + \sqrt{\frac{\rho}{\rho_1}}\right)} = 0$$

Its solution with respect to v_s/v_0 gives

$$\frac{v_s}{v_0} = \frac{1}{2}\sqrt{\frac{\rho}{\rho_1}}\left[1 + \sqrt{1 + \frac{2\sigma_p}{1 + \sqrt{\rho_1/\rho}}}\right] \tag{12}$$

3. Efficiency of the Screw. We are going now to determine the ideal efficiency of a cavitating propeller. The useful power developed by the screw is, obviously,

$$L_u = Pv_0 \tag{13}$$

The power expended by the screw L_e is obtained from the increment of kinetic energy of a flow of fluid crossing the screw in unit time

$$L_e = m\frac{v_\infty^2}{2} - m\frac{v_0^2}{2} \tag{14}$$

Here m is the mass of the fluid worked up by the screw

$$m = \rho F_p v_s \tag{15}$$

Substituting m in (14) and making use of (6) we find

$$L_e = \rho F_p v_s (v_\infty - v_0)\left(\frac{v_\infty + v_0}{2}\right) = P\frac{v_\infty + v_0}{2} \tag{16}$$

The ideal efficiency of a cavitating screw propeller η_i is equal to the ratio of useful power to expended power

$$\eta_i = \frac{L_u}{L_e} = \frac{Pv_0}{P\frac{v_\infty + v_0}{2}} = \frac{2}{1 + \frac{v_\infty}{v_0}} \tag{17}$$

Substituting in (17) the expressions (9) and (12) we determine the efficiency as a function of ρ/ρ_1 and σ_p

$$\eta_i = \frac{2}{\left(1 - \sqrt{\frac{\rho}{\rho_1}}\right) + \frac{v_s}{v_0}\left(\frac{\rho}{\rho_1} + \sqrt{\frac{\rho}{\rho_1}}\right)} =$$

$$= \frac{2}{\left(1 - \sqrt{\frac{\rho}{\rho_1}}\right) + \frac{1}{2}\left(\sqrt{\frac{\rho}{\rho_1}} + 1\right)\left[1 + \sqrt{1 + \frac{2\sigma_p}{1\sqrt{\rho/\rho_1}}}\right]} \tag{18}$$

In the particular case of $\rho/\rho_1 = 1$ we obtain the well-known formulae of the Rankine-Froude theory of the ideal propeller.

4. The calculations made according to the formulae we derived here have given the following results:

a) With the thrust coefficient σ_p given, the relative velocity of the water inflow to the screw v_s/v_0 decreases with the ratio ρ_1/ρ. The decrease is especially sharp with small values of ρ_1/ρ. For certain values of σ_p and ρ_1/ρ the relative velocity v_s/v_0 can take values below unity.

b) For a given thrust coefficient σ_p the relative change of pressure $2\Delta p/\rho v_0^2$ falls off abruptly with decreasing ρ_1/ρ. The decrease is most conspicuous with large thrust coefficients and a small value of ρ_1/ρ. For certain values of σ_p and ρ_1/ρ the pressure change in the screw cross-section can take negative values. As the thrust coefficient is diminished, the pressure change becomes negative at increasing values of ρ_1/ρ. The condition for the appearance of a negative change in pressure may be expressed as follows

$$\frac{1}{2}\sqrt{\frac{\rho_1}{\rho}}\left[1+\sqrt{1+\frac{2\sigma_p}{1+\sqrt{\rho_1/\rho}}}\right] < 1$$

c) For a fixed thrust coefficient σ_p the efficiency of an ideal propeller decreases with ρ_1/ρ. The decrease is especially pronounced with small values of ρ_1/ρ.

It may be remarked in conclusion that the above calculations are of somewhat arbitrary nature, since in reality the density of the flow cast by a cavitating propeller depends upon the working regime of the latter. The question as to the relation between ρ_1/ρ and σ_p must be subjected to a special investigation.

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REFERENCES

[1] V. L. Posdunine, C. R. Acad. Sci. URSS, XXXIX, No. 8 (1943).

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THEORY OF ELASTICITY

MOTION OF SEISMIC ENERGY IN DIFFERENT MEDIA

By V. G. GOGOLADZE

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The problem of the total inner reflection of elastic waves has been solved for the half-space in a most complete manner, owing to the work done by Lamb (1), Sobolev (2) and Smirnov (3). In the field of the theory of seismic prospecting particular interest attaches to the study of the motion of seismic energy in different media meeting along a plane. If the reflected and refracted plane waves are real (homogeneous), their energy motion is determined by means of rays normal to the fronts of the plane waves. If, on the other hand, some of the waves are transformed into complex (unhomogeneous) plane waves, seismic geometry finds itself unable to analyse their energy motion, and so it is with the Rayleigh waves which belong in the class of complex waves, too (in this case we have to deal with complex rays).

In the present paper which has the seismic energy motion of complex waves for its subject, the notion of «ray» is replaced by that of «energy flow line».

Let us consider two different elastic media meeting along the plane OXZ. The velocities of the longitudinal waves in $y>0$ and $y<0$ are a and c, respectively; and the velocities of the transverse waves are b and d. We suppose them to satisfy the inequality $d<b<a<c$ (the other possible cases can be dealth with in an analogous manner). Suppose that in $y>0$ a longitudinal wave with potential φ falls on the boundary $y=0$

$$\varphi=\varphi\left(t-\theta x+y\sqrt{\frac{1}{a^2}-\theta^2}\right)=\operatorname{Re} f\left(t-\theta x+y\sqrt{\frac{1}{a^2}-\theta^2}\right) \tag{1}$$

where f is a regular function in the upper complex half-plane.

At the boundary $y=0$ the vectors of displacement and stress are supposed to be continuous. Then (4) the potentials of reflected and refracted waves for $1/a>|\theta|>1/c$ may be written in the following form

$$\left.\begin{aligned}\varphi_1&=\operatorname{Re} D_1 f\left(t-\theta x-y\sqrt{\frac{1}{a^2}-\theta^2}\right)\\ \psi_1&=\operatorname{Re} D_2 f\left(t-\theta x-y\sqrt{\frac{1}{b^2}-\theta^2}\right)\end{aligned}\right\}\ y>0$$

$$\left.\begin{aligned}\varphi_2&=\operatorname{Re} D_3 f\left(t-\theta x-iy\sqrt{\theta^2-\frac{1}{c^2}}\right)\\ \psi_2&=\operatorname{Re} D_4 f\left(t-\theta x+y\sqrt{\frac{1}{d^2}-\theta^2}\right)\end{aligned}\right\}\ y<0 \tag{2}$$

where the $D_k\,(k=1,\ 2,\ 3,\ 4)$ are intensity coefficients of the reflected and refracted waves (5).

The plane front of the potential φ_2 is determined by the equality

$$t-\theta x-iy\sqrt{\theta^2-\frac{1}{c^2}}=0 \quad \text{or} \quad t-\theta x=0,\quad y=0 \tag{3}$$

so it appears as if the ray was directed along the boundary $y=0$. As a matter of fact, in the case of complex waves the rays (*i. e.* the energy trajectories) are curved lines in a homogeneous medium, and their direction varies with time.

The energy flux $\vec{S}(S_x, S_y)$ of a refracted complex longitudinal wave φ_2 is determined ([6]) by the formulae

$$S_x = -\left[\frac{\partial u}{\partial t}\tau_{xx} + \frac{\partial v}{\partial t}\tau_{xy}\right], \quad S_y = -\left[\frac{\partial u}{\partial t}\tau_{xy} + \frac{\partial v}{\partial t}\tau_{yy}\right] \tag{4}$$

where

$$u = \frac{\partial \varphi_2}{\partial x}, \quad v = \frac{\partial \varphi_2}{\partial y}, \quad \tau_{xx} = \mu_1\left[\left(\frac{c^2}{d^2} - 2\right)\Delta\varphi_2 + 2\frac{\partial^2 \varphi_2}{\partial x^2}\right]$$

$$\tau_{xy} = 2\mu_1 \frac{\partial^2 \varphi_2}{\partial x\, \partial y}, \quad \tau_{yy} = \mu_1\left[\left(\frac{c^2}{d^2} - 2\right)\Delta\varphi_2 + 2\frac{\partial^2 \varphi_2}{\partial y^2}\right]$$

μ_1 is shearing modulus in $y < 0$. Denote

$$D_3 f''\left(t - \theta x - iy\sqrt{\theta^2 - \frac{1}{c^2}}\right) = \omega_1(t - \theta x, y, \theta) + i\omega_2(t - \theta x, y, \theta) \tag{5}$$

After simple computations we obtain a rather curious result for S_x

$$S_x = \frac{\theta\mu_1}{d^2}\left[\omega_1^2 + 2d^2\left(\theta^2 - \frac{1}{c^2}\right)(\omega_1^2 + \omega_2^2)\right] > 0, \quad \theta > 0 \tag{6}$$

Therefore the component of the energy flux along the OX-axis has a constant direction. For S_y we obtain the following expression

$$S_y = -\frac{\mu_1}{d^2}\sqrt{\theta^2 - \frac{1}{c^2}}\,\omega_1\omega_2 \tag{7}$$

For $\theta = 1/a$ (directed along the boundary $D_3 = 0$) and $\theta = 1/c$ (total inner reflection) we have $S_y = 0$. In all the other cases the S_y-component of the energy flux can in virtue of (7) change sign in the course of time.

The energy flux $\vec{S}(S_x, S_y)$ can enter as well as leave the medium $y < 0$. The energy of an unhomogeneous wave can overflow the separation boundary $y = 0$ at different places and different times. Because of the constancy of the sign of S_x and the variation of S_y, unhomogeneous waves are intimately linked up with the separation boundary, where their effect is a maximum. The projection of a line of energy flow on to the OX-axis gives a spurious ray normal to the front (3). In the case of periodical optical waves satisfying Maxwell's equation, this circumstance was discovered by Eichenwald ([7]).

The energy trajectory is defined by the equation

$$\frac{dy}{dx} = -\frac{\sqrt{\theta^2 - \frac{1}{c^2}}\,\omega_1\omega_2}{\theta\left[\omega_1^2 + 2d^2\left(\theta^2 - \frac{1}{c^2}\right)(\omega_1^2 + \omega_2^2)\right]} \tag{8}$$

A similar computation of the energy flow of a homogeneous transverse wave shows that the vector $\vec{S}(S_x, S_y)$ in this case has a constant direction, and the energy of the wave does not leave the medium $y < 0$.

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REFERENCES

[1] H. Lamb, Phil. Trans. (203) A (1904). [2] С. Соболев, Тр. Сейсмолог. ин-та АН СССР, No. 7 (1930). [3] С. Соболев и В. Смирнов, там же, № 20 (1932). [4] V. Gogoladze, C. R. Acad. Sci. URSS, XLIX, No. 5 (1945). [5] V. Gogoladze, ibid., XLIX, No. 6 (1945). [6] A. E. Лява, Математическая теория упругости. [7] А. Эйхенвальд, Ann. der Physik, 35 (1911); ЖРФХО, 41, 131 (1909).

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PHYSICS

ON THE THEORY OF THE LIGHT FIELD IN A SCATTERING MEDIUM

By A. GERSHUN

(Communicated by S. I. Vavilov, Member of the Academy, 13. IV. 1945)

Denote by B the brightness of rays confined within elementary solid angles $d\omega$, and let the quantity

$$\mathcal{E} = \int_{(4\pi)} B d\omega$$

be described as space illumination. Here (4π) denotes integration over all directions in space. The quantity $\mathcal{E}$ is proportional to the volume density of the light energy(1). An elementary volume dv of a medium absorbs the luminous flux $k\mathcal{E}dv$ and scatters the luminous flux $\rho\mathcal{E}dv$, where k is the absorption coefficient and ρ is the scattering coefficient. Direct light travelling the path s, without changing direction, is proportional to $e^{-(k+\rho)s}$.

The difference between the luminous fluxes falling on either side of the surface may be considered as the flux of the vector

$$\vec{E} = \int_{(4\pi)} B\vec{d\omega}$$

where $\vec{d\omega}$ is a vector directed along the axis of and equal to an elementary solid angle $d\omega$. The projection of the vector $\vec{E}$ on some direction gives the difference between the illumination of the two sides of the surface element perpendicular to this direction. According to the law of conservation of energy

$$\operatorname{div}\vec{E} = -k\mathcal{E}$$

In deep layers of a turbid medium where the illumination is due to the scattered light, the relative distribution of the brightness values along the different directions of the space (and therefore the distribution of the illumination of elements having different orientation) may be considered invariable. The admissibility of such an approximation follows from the theoretical consideration of the problem(3), as well as from experimental data(4).

Assume that the medium is homogeneous, the light field symmetrical with respect to the vertical axis, and its characteristics equal in all points lying at the same depth z. Denote by $B(\vartheta, z)$ the brightness which the ray forming an angle ϑ with the vertical ($\vartheta = 0$ corresponds to the direction downwards) has at some point at a depth z. Then

$$\mathcal{E}(z) = \mathcal{E}_1(z) + \mathcal{E}_2(z), \quad |\vec{E}(z)| = E_1(z) - E_2(z)$$

where $\mathcal{E}_1(z) = 2\pi \int_{\vartheta=0}^{\vartheta=\frac{\pi}{2}} B(\vartheta, z)\sin\vartheta\, d\vartheta$ is space illumination by the light coming

from above; $\mathcal{E}_2(z) = 2\pi \int_{\theta=\frac{\pi}{2}}^{\theta=\pi} B(\theta, z) \sin\theta\, d\theta$ is space illumination by the light coming from below; $E_1(z) = 2\pi \int_{\theta=0}^{\theta=\frac{\pi}{2}} B(\theta, z) \cos\theta \sin\theta d\theta$ is horizontal illumination from above; $E_2(z) = -2\pi \int_{\theta=\frac{\pi}{2}}^{\theta=\pi} B(\theta, z) \cos\theta \sin\theta\, d\theta$ is horizontal illumination from below.

Assuming in accordance with the initial approximation $B(\theta, z) = b(\theta) f(z)$, and taking into account the vertical symmetry, we find from

$$\operatorname{div} \int_{(4\pi)} B\vec{d}\omega = -k \int_{(4\pi)} B d\omega \text{ that } \frac{dE_1(z)}{dz} - \frac{dE_2(z)}{dz} = \frac{f'(z)}{f(z)} [E_1(z) - E_2(z)] = -k [\mathcal{E}_1(z) + \mathcal{E}_2(z)]$$

Hence follows that $f(z) = Ce^{-Lz}$, where C is a constant determined by boundary conditions, L is determined by the photometric properties of the medium, and

$$L[E_1(z) - E_2(z)] = k[\mathcal{E}_1(z) + \mathcal{E}_2(z)]$$

The assumed invariability of the spatial distribution of brightness leads naturally to the exponential law of attenuation of light with increasing depth.

For estimating the attenuation factor L, which characterizes the rate of attenuation of the light as the depth increases, it is convenient to introduce the auxiliary values independent in this approximation of the depth. These are

$$R = \frac{E_2(z)}{E_1(z)}, \quad g_1 = \frac{\mathcal{E}_1(z)}{E_1(z)}, \quad g_2 = \frac{\mathcal{E}_2(z)}{E_2(z)}$$

The quantity R may be considered as the reflection coefficient of the medium, and quantities $1/g_1$ and $1/g_2$ as mean values of the cosines of the angles to the vertical for the light coming from above and from below. These latter values are estimated with reference to brightness. If the light falls in a parallel beam normal to the layer, the coefficient $g = 1$. If the light is isotropic and the rays fall in all directions and have the same brightness, then $g = 2$. Using these quantities we obtain

$$L = \frac{g_1 + g_2 R}{1 - R} k$$

Numerous measurements have shown that in seas and lakes the reflection coefficient R of the water in bulk never exceeds a few per cent. Hence

$$L = g_1 k$$

The characteristic curve of scattering in sea water is so much extended forward that the attenuation of daylight with increasing depth is determined almost completely by the true absorption.

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REFERENCES

[1] А. А. Гершун, Световое поле, Л., 1936. [2] А. А. Гершун, Тр. ГОИН, XI, 99, 3 (1936). [3] V. Ambarzumian, Bull. Acad. Sci. URSS, sér. géogr. et géophys., No. 3 (1942). [4] L. V. Whitney, J. Opt. Soc. America, 31, 12, 714 (1941).

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PHYSICS

ON METHOD IN SUPERSONIC ABSORPTION MEASUREMENTS

By E. J. PUMPER

(Communicated by N. D. Papalexi, Member of the Academy, 28. II. 1945)

For determining supersonic absorption in gases most authors employed supersonic interferometer. Measurements with this device, however, as has been shown earlier ([1—3]), involve errors due to non-uniformity of the ultrasonic fields radiated by piezo-electric quartz crystals. Corrections have to be introduced to reduce these errors in magnitude. Even then, however, the measurements are not accurate enough to solve certain problems in physics involving a determination of the relaxation time of intramolecular movements. Furthermore, the corrections we have introduced cannot be determined in the dispersion region ([4—6]).

An elaborate theory of the interferometers has been evolved by Krasnushkin ([7]), who gives a method for determining the corrections more precisely. Unfortunately, however, the range of application of his method is limited. He has not carried the discussion beyond the simple case $\alpha l \gg 1$*. Yet, even with $\alpha l = 2$ in the air at atmospheric pressure and a frequency of 500 kilocycles we have $l = 40$ cm. With a decrease in pressure, l decreases $^1/_p$ times (p being the pressure in atmospheres). But, according to our curves [([3]), page 419, Fig. 6], under these conditions the reactions are already so small that the errors in readings may essentially decrease the precision of the measurements.

In light gases, such as helium, for example, the reactions of the reflector on the quartz are in general very weak. Absorption of sound in helium is less than in air. Hence l should be greater in helium than in air. Therefore there is a great deal of trouble in applying the given theory here.

It would be convenient to have such a method in which the corrections for scattering of the field, radiated by the piezo-electric quartz crystal, would in general be small. From this standpoint the supersonic interferometer is especially unfavourable, owing to the fact that a standing wave is set up in it. Readings of the reactions are made at resonance positions of the reflector. These readings are especially sensitive to the phase with which the reflected wave returns to the radiator, but the phase may essentially change from a slight scattering of the ultrasonic beam. That is why slight non-uniformities in the ultrasonic field may noticeably distort the readings of the interferometer.

The correct thing to do would be to measure the diminution in intensity of the supersonic beam in a progressive wave, where the phase effects do not play any rôle and the small scatterings of the field should make themselves felt to a lesser extent. In that case a supersonic field with slight non-uniformities may be regarded as a uniform field.

* α is the coefficient of absorption per unit length, and l is the distance from the reflector to the quartz crystal.

This is confirmed by Korolev's experiments ([8]) in air and a number of researches of Bazhulin ([9]) in liquids. Both these authors measured supersonic absorption by the Toepler method. Without introducing any corrections for supersonic scattering, Korolev got values in air similar to mine. Bazhulin likewise obtained quite reliable results in liquids.

Application of the Toepler method to gases is likewise limited, however, owing to its insufficient sensitivity at small densities*. Furthermore, it involves photometric analysis of the photographs, which is rather cumbersome.

It would be convenient to have such a method in which an electric indicator whose oscillation could be amplified served as the supersonic receiver. This method would be particularly effective if the frequency of the electric effect in the receiver differed from the supersonic frequency. Then it would be easily possible to distinguish the effects caused by supersonic radiation and by the electric induction of the radiator to the receiver.

Such a method may be realized if a supersonic radiation modulated by a low frequency is employed for the measurement. To this end an external EMF modulated by a harmonic acoustic tone is applied to the radiating plate of the piezoelectric quartz crystal. Then the potential v on the latter will be

$$v = v_0 (1 + M \cos \Omega t) \sin \omega t$$

where ω is the supersonic frequency, and Ω is the modulation frequency.

The supersonic radiation will likewise be modulated if the entire modulation spectrum of the electric action will lie within the bounds of the band-pass of the resonance curve of the quartz plate. This may be attained by artificially increasing the decrement of damping of the quartz by pressing the plate.

It is known that when a sound falls on a solid wall it undergoes ponderomotive pressure proportional to the density of the energy of acoustic flow, *i. e.* to the square of the amplitude of acoustic displacement ([10]). If this relation established by Rayleigh is applied to the case of a modulated supersonic radiation, then the displacement ξ_l at a distance l from the quartz will be

$$\xi_l = \xi_{0l} (1 + M \cos \Omega t) \sin \omega t$$

and will give rise to a pressure

$$P = k\xi_{0l}^2 (1 + M \cos \Omega t)^2 = k\xi_{0l}^2 [1 + {}^1/_2 M^2 + 2M \cos \Omega t + {}^1/_2 M^2 \cos 2\Omega t]$$

i. e., the pressure on the wall will have a modulation frequency component, proportional in magnitude to the square of the amplitude of the carrying sound wave**.

If the membrane of a microphone is the solid wall on which falls the supersonic radiation, the latter becomes the source of an EMF of frequency Ω with an amplitude proportional to ξ_{0l}^2. In addition, an EMF of frequency 2Ω arises. To reduce its influence and also the influence of outside noises, it helps to have a resonance filter of frequency Ω after the microphone. Besides, a block capacity must be inserted in the microphone circuit to suppress the electric induction of the radiator in the region of high frequencies.

After filtration, the oscillations may be amplified. To prevent a standing wave from forming in the acoustic field investigated, the membrane of the microphone should be mounted at an angle φ to the front of the incident wave. An absorber should be set up perpendicular to the direction of the reflected wave.

It is easy to show that the dependence of the EMF that arises therewith in the microphone upon l remains exponential with a damping index 2α. If the EMF of the microphone is applied after filtration to a linear amplifier, the output vol-

* At the basis of the method lies the condition that the non-uniformities of the optical system are small in comparison with the non-uniformities of the medium investigated.

** A very similar case, *viz.* that of modulated supersonic action on a solid sphere fixed on a string, was analysed theoretically by S. M. Rytov.

tage will likewise be of the frequency Ω. The amplitude of the output voltage will be

$$v_{0l} = v_0 e^{-2al}$$

To measure the damping of ultrasonic radiation the customary method may be employed, which holds for the measurement of absorption in a plane progressive wave. Measuring the output voltage when the microphone is at two different distances from the quartz, we get:

$$v_1 = v_0 e^{-2al_1}, \qquad v_2 = v_0 e^{-2al_2}$$

$$\frac{v_1}{v_2} = e^{2a(l_2 - l_1)}, \qquad a = \frac{1}{2(l_2 - l_1)} \lg \frac{v_1}{v_2}$$

A qualitative test of this method was made in 1941 by V. A. Petrova, a postgraduate student, and all the expected effects were obtained. Her untimely death in the spring of that same year interrupted quantitative research on the method.

The method may be expected to meet with its most favourable conditions in liquids, where the piezo-electric quartz crystal undergoes strong damping. Thanks to this it possesses a large band-pass, and modulation of the supersonic radiation in the liquid presents no problem at all. The first quantitative experiments with supersonic radiation in a liquid by this method have been carried out by A. P. Kapustin. They, too, gave the expected result.

It is of interest to compare the sensitivity of this method with the Toepler method that is customarily employed to measure absorption in liquids.

The present paper is one of a series of supersonic researches started by the late L. I. Mandelstam.

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REFERENCES

[1] A. T. Dadajan and E. J. Pumper, C. R. Acad. Sci. URSS, XX, No. 7—8 (1938). [2] P. E. Krasnushkin and E. J. Pumper, ibid., XXIII, No. 5 (1939). [3] E. J. Pumper, J. of. Physics USSR, **1**, 5—6, 411 (1939). [4] W. H. Pielemeier, J. Acoust. Soc., **15**, 1, 22 (1943). [5] W. H. Pielemeier and W. H. Byers, J. Acoust. Soc., **15**, 1, 17 (1943). [6] W. H. Byers, J. Chem. Phys., **11**, 8, 348 (1943). [7] П. Е. Краснушкин, Уч. зап., МГУ, 74, физика, 73 (1944); Phys. Rev., **65**, No.5—6, 190 (1944). [8] F. A. Korolev, C. R. Acad. Sci. URSS, XX, No. 7—8 (1938). [9] P. V. Bazhulin, ibid., XIX, No. 3 (1938). [10] Lord Rayleigh, Sci. Papers, **5**, 41 (1912).

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GEOPHYSICS

ON THE THEORY OF DIFUSION OF BODIES OF FINITE DIMENSIONS IN A TURBULENT ATMOSPHERE

By M. I. YUDIN

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The first theoretical investigations of the process of diffusion of bodies in a turbulent atmosphere were based on the assumption of a complete analogy between the processes of molecular and turbulent diffusion. L. Richardson even suggested the following formula for determining the eddy diffusivity k from the diffusion of falling particles

$$2kT = \sigma^2(T) - \sigma^2(0) \tag{1}$$

Here $\sigma^2(T)$ is the dispersion of particles at the instant T. His own experiments, however, showed ([1]) that the coefficient k thus determined fails to characterize the turbulence in a unique manner, but rather depends upon various external factors, in the first place, upon the spatial scale of the phenomenon. Theoretically, the influence of the scale has been explained by Obukhov ([2]).

It is the object of the present paper to show that even if the experiment is arranged in such a way that the influence of the scale on the dispersion is eliminated (the conditions under which this may be achieved will be dealt with elsewhere), formula (1) does not determine the properties of the turbulence. There will also be brought to light the operation of a new factor in the diffusion of particles in the atmosphere, *viz.* the velocity with which they fall.

Let us consider the simplest case of a steady fall of a solid sphere of radius R in a homogeneous turbulent atmosphere under invariable external conditions. We will confine the discussion to Reynolds' numbers lying below the critical value. The direction of the coordinate axes we define as follows: the x-axis coincides with the mean velocity of the wind $\vec{U}$, the z-axis runs vertically downward.

Denote the components of the wind velocity $\vec{U} + \vec{u}(x, y, z, t)$ by $U + u_x(x, y, z, t)$, $u_y(x, y, z, t)$ $u_z(x, y, z, t)$; the absolute velocity of the sphere, by $\frac{\vec{dr}}{dt}\left(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\right)$; its relative velocity, by $\vec{v}(v_x, v_y, v_z)$; the force of resistance of the air, by $-G_R(v)\vec{v}$; mg is gravity.

Let the critical velocity of fall v_k be defined by the relation

$$G_R(v_k)\, v_k = mg \tag{2}$$

The quantities $\frac{v_x}{v_k}$, $\frac{v_y}{v_k}$, $\frac{v_z - v_k}{v_k}$ * being assumed to be small, it is possible

* It is well to note that this assumption is not necessary for small v_k, when $G_R(v)$ is altogether independent of v, which considerably extends the range of applicability of equation (3).

to represent the equations of motion in the form

$$\left.\begin{aligned} \frac{d^2x}{dt^2} &= -\frac{g}{v_k}\left(\frac{dx}{dt} - U - u_x\right) \\ \frac{d^2y}{dt^2} &= -\frac{g}{v_k}\left(\frac{dy}{dt} - u_y\right) \\ \frac{d^2z}{dt^2} &= -\frac{ng}{v_k}\left(\frac{dz}{dt} - v_k - u_z\right) \end{aligned}\right\} \tag{3}$$

where

$$1 + \frac{v_k}{G_R(v_k)}\left.\frac{dG_R(v)}{dv}\right|_{v=v_k} = n. \tag{4}$$

The following physical scheme appears to be the simplest way to present the problem. A ball is dropped from the point $x=0$, $y=0$, $z=0$ with an initial velocity v_k. This experiment is repeated many times with identical balls, and each time the position of the ball in space is recorded T seconds after the thraw. Then owing to the invariability of external conditions, the distribution density of the balls in the space will tend to some limiting function of distribution.

During each experiment the values of the components of the vector $\vec{u}$ are to be considered as random quantities correlated in space and time, whose mathematical expectation is 0. Counting the time in each experiment from the instant we cast the ball, we shall obtain from equations (3) the expressions for the random values of the coordinates of the centre of the ball at instant T

$$\left.\begin{aligned} x(T) - UT &= \int_0^T u_x(\tau)\left[1 - e^{-\frac{g}{v_k}(T-\tau)}\right]d\tau \\ y(T) &= \int_0^T u_y(\tau)\left[1 - e^{-\frac{g}{v_k}(T-\tau)}\right]d\tau \\ z(T) - v_k T &= \int_0^T u_z(\tau)\left[1 - e^{-\frac{ng}{v_k}(T-\tau)}\right]d\tau \end{aligned}\right\} \tag{5}$$

From this we find the dispersion of the balls in the form of double integrals in quantities which, following Keller and Friedmann [3], we shall call «correlation moments».

Let us consider some simplified expressions of the dispersions for sufficiently large T, $T \gg v_k/g$, when the second terms in brackets may obviously be omitted

$$\left.\begin{aligned} \sigma_x^2(T) &= \overline{[x(T) - UT]^2} = 2\int_0^T d\tau \int_0^\tau \overline{u_x(\tau)\,u_x(\tau_1)}\,d\tau_1 \\ \sigma_y^2(T) &= \overline{y^2(T)} = 2\int_0^T d\tau \int_0^\tau \overline{u_y(\tau)\,u_y(\tau_1)}\,d\tau_1 \\ \sigma_z^2(T) &= \overline{[z(T) - v_k(T)]^2} = 2\int_0^T d\tau \int_0^\tau \overline{u_z(\tau)\,u_z(\tau_1)}\,d\tau_1 \end{aligned}\right\} \tag{6}$$

Expressions (6) are quite analogous to the well-known relation of Taylor [4], which by means of formal transformations was further reduced to the form $2kT$. A more detailed analysis shows, however, that equations (6) are far from being an explicit expression of the quantities σ_x^2, σ_y^2, σ_z^2. As a matter of fact, the ball which at the instant τ_1 finds itself at the point

$x(\tau_1), y(\tau_1), z(\tau_1)$ will have its position by the time $\tau = \tau_1 + \vartheta$ the more indeterminate, the larger the quantities $\sigma_x(\vartheta)$, $\sigma_y(\vartheta)$, $\sigma_z(\vartheta)$. This means that the correlation moments, too, must depend upon $\sigma_x(\vartheta)$, $\sigma_y(\vartheta)$, $\sigma_z(\vartheta)$. The existence of a correlation between dx/dt and u_x does not in principle remove the above dependence.

It is easy to see that expressing the above considerations in mathematical terms, we obtain a system of three non-linear integral equations for the functions σ_x, σ_y, σ_z. This system is solvable for small T because then the quantity $\overline{u_x(\tilde{x}, y, z, t)\, u_x(\tilde{x} + \sigma_x(\vartheta)\xi, y + \sigma_y(\vartheta)\eta, z + v_k\vartheta + \sigma_z(\vartheta)\zeta, t + \vartheta)}$ and those similar to it can be represented by the first terms of a Taylor series. (Here the substitution $\tilde{x}(t) = x(t) - Ut$ has been made, and ξ, η, ζ are dimensionless variables practically confined in absolute magnitude to several units).

With this $\sigma_x(T)$, $\sigma_y(T)$, $\sigma_z(T)$ will be found to depend not merely upon the values of the partial derivatives of the correlation moment (turbulence characteristics of the medium), but also upon the velocity of fall v_k, diminishing as the latter grows.

For large T (which case is of the greatest interest in this formulation of the problem) no general solution can be obtained. However, the conclusion that the coefficient k as determined by formula (1) is not a physical characteristic of turbulence of the air, but rather a quantity dependent upon external factors, does not by any means lose its validity.

Of considerable interest is the case of large v_k, which can be explored to the end. In fact, with increasing v_k there is a decrease in the influence which the particular form of the trajectory of an individual particle has on the products from which the correlation moments are made up. If a new variable of integration be introduced $\lambda = v_k\vartheta$, it will at once be apparent that the following approximate equalities hold for sufficiently large v_k

$$\sigma_x^2(T) = \frac{2}{v_k}\int_0^T d\tau \int_0^{v_k(T-\tau)} \overline{u_x(\tilde{x}(\tau), y(\tau), z(\tau), \tau)\, u_x(\tilde{x}(\tau), y(\tau), z(\tau) + \lambda, \tau)}\, d\lambda \qquad (7)$$

Practically, the relation between the values of instantaneous velocity at two different heights is rapidly extinguished as the difference between the heights grows. Therefore when T is so large that

$$\frac{\int_0^{\alpha v_k T} \overline{u_x(\tilde{x}, y, z, \tau)\, u_x(\tilde{x}, y, z+\lambda, \tau)}\, d\lambda}{\int_{\alpha v_k T}^{\infty} \overline{u_x(\tilde{x}, y, z, \tau)\, u_x(\tilde{x}, y, z+\lambda, \tau)}\, d\lambda} \gg 1 \quad (\alpha \ll 1) \qquad (8)$$

then, introducing the notation

$$c_x T = \int_0^T d\tau \int_0^{\infty} \overline{u_x(\tilde{x}(\tau), y(\tau), z(\tau), \tau)\, u_x(\tilde{x}(\tau), y(\tau), z(\tau) + \lambda, \tau)}\, d\lambda \qquad (9)$$

etc., we obtain quite simple formulae

$$\sigma_x^2(T) = \frac{2c_x T}{v_k}, \quad \sigma_y^2(T) = \frac{2c_y T}{v_k}, \quad \sigma_z^2(T) = \frac{2c_z(T)}{v_k} \qquad (10)$$

The quantities c_x, c_y, c_z whose dimensionality is $[L^3T^{-2}]$ can obviously be regarded as representative mean characteristics of turbulence in our problem (where the «scale influence» is absent). Their close connexion with the process of heat transfer, momentum, *etc.* is apparent.

The available experiment data agree satisfactorily with the dependence of the diffusion upon the critical velocity and the time of fall of the balls, as expressed by formulae (10), for $v_k \geqslant 3$ m/sec. Yet a more extensive verification by experiment is desirable.

To end with, it may be remarked that our conclusions admit of a simple generalization to the case of unhomogeneous medium if a) the relative variations of all mean values within the limits of «the characteristic distance» v_k^4/g are small, and if b) there holds the inequality $T \gg \sqrt{\mu}\,(v_k/g)$, where μ is the highest value of the ratio $T\overline{u_s^2(\tau)} \Big/ \int_0^T \overline{u_s^2(\tau)}\,d\tau$ for $0 \leqslant \tau \leqslant T$ and $s = x, y, z$.

In particular, the formula (10) is valid if the conditions a) and b) are satisfied. In this case the quantities c_x, c_y, c_z may be interpreted as mean weighed turbulence characteristics of a layer extending in thickness from $z = 0$ to $z = v_k T$.

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REFERENCES

[1] L. F. Richardson, Proc. Roy. Soc., A, **110** (1926). [2] A. M. Obukhov, Bull. Acad. Sci. URSS, série géogr. et géophys., No. 4—5, 453 (1941). [3] L. Keller and A. Friedmann, Proc. First Intern. Congr. f. Appl. Mech., 395 (1924). [4] G. J. Taylor, Proc. Lond. Math. Soc., **20**, 196 (1922).

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TECHNICAL PHYSICS

THE SOUND OF A MOVING AEROPLANE PROPELLER

By M. F. SHIROKOV

(Communicated by B. A. Vvedensky, Member of the Academy, 30. I. 1945)

The theoretical solution of the problem to be discussed here does not appear to have been attempted by any one except Hart (1) who, using the existing solution for a propeller at rest, modified it by making allowance for the influence of the wind. The movement of an aeroplane in the medium he considered to be equivalent to the movement of the medium past a resting aeroplane. According to him, the sound characteristic (diagram of sound pressure amplitude distribution in space) is displaced in the direction of flight. This is an erroneous conclusion, however, and it will be shown later that actually the displacement is in the opposite sense to the flight.

It should be noted that the vibrations generated by a region of sound sources such as is formed by an aeroplane propeller, show interference of considerable phase difference at the point of observation, because different sources vibrate with different phases and because the dimensions of the propeller are comparable to the length of the sound wave. Therefore, in the case of a moving propeller we have to look for a new solution of the differential equation of acoustics, proceeding upon general relations for moving sound sources as established in my earlier paper (2)*.

Define the x-axis to be directed along the rotation axis of the propeller and proceed then to a cylindrical system of coordinates

$$x = x, \quad y = R\cos\varphi, \quad z = R\sin\varphi \tag{1}$$

The field of force revolving with angular velocity ω, while it is moving along the x-axis with velocity v, may be written as follows

$$\vec{K} = \vec{K}(x - vt, R, \varphi - \omega t) \tag{2}$$

Here $\vec{K}$ is the force acting on a unit volume of the medium at a point (x, R, φ). Being a periodical function of the parameter $\chi = \varphi - \omega t$, it can be developed in a Fourier series

$$\vec{K} = \sum_{m=0}^{m=\infty} K_m(x - vt, R)\, e^{-imn(\varphi - \omega t)} \tag{3}$$

where n is the number of propeller blades.

It may be seen from this formula that the field of force of a propeller moving at once along and about its axis can be looked upon as a superposition of fields of type (2a) that oscillate harmonically as they move forward. Accordingly, for the sound pressure of the m-th harmonic relations (7a) and (8a) may be applied. Prior to using them, however, we must write down the

* References to the formulae of that paper are marked by letter «a». The symbols used to denote the physical quantities are throughout the same as before.

expressions for the components of the force $\overrightarrow{K}$, which, according to the hydrodynamic theory of propellers, are reduced to two forces K_φ and $-K_x$ ($K_\varphi > 0$ and $K_x > 0$), perpendicular to each other and directed opposite to the force of resistance to the rotary motion of the propeller and to the force of traction, respectively. In virtue of (3) we have

$$\left.\begin{aligned} K_\varphi &= \sum_{m=0}^{m=\infty} K_\varphi^m (x - vt, R)\, e^{-imn(\varphi-\omega t)} \\ K_x &= \sum_{m=0}^{m=\infty} K_x^m (x - vt, R)\, e^{-imn(\varphi-\omega t)} \end{aligned}\right\} \qquad (4)$$

Suppose further that the terms containing the derivatives $r'^{-2}\left(1-\frac{v}{c}\cos\vartheta'\right)^{-1}$ with respect to the coordinates can be discarded on account of smallness, and that K_φ^m and K_x^m are distinct from zero within the volume of a disk of small thickness $d = x_2 - x_1$ and of radius R_0 equal to the radius of the propeller. The distance from the centre of the propeller to the observation point P at an instant t' will be denoted by r'_0.

Then, by means of (7a), (8a), (5a) and (1) we find the expression for the sound pressure of the m-th harmonic

$$P_m = \frac{mn\omega i^{(mn+1)}\, e^{imn\omega\left(t-\frac{r'}{c}\right)}}{2\pi r'_0\left(1-\frac{v}{c}\cos\vartheta'\right)} \int_0^{R_0} R\left(\frac{cX_\varphi^m}{\omega R} - \frac{X_x^m\cos\vartheta'}{1-\frac{v}{c}\cos\vartheta'}\right) J_{mn}\left(\frac{mn\omega}{c} R\sin\vartheta'\right) dR \qquad (5)$$

where

$$J_z(z) = \frac{1}{2\pi i^k}\int_0^{\pi} e^{iz\cos\varphi - imn\varphi}\, d\varphi$$

is a Bessel function of the first kind of order k

$$\left.\begin{aligned} X_\varphi^m &= \int_{x_1}^{x_2} K_\varphi^m (x - vt, R)\, dx = X_\varphi^m(R) \\ X_x^m &= \int_{x_1}^{x_2} K_x^m (x - vt, R)\, dx = X_x^m(R) \end{aligned}\right\} \qquad (6)$$

Putting in (5) $v = 0$ and comparing the result thus obtained with (5), we come to solve the question as to the corrections that have to be introduced into the formula for the sound pressure of a resting propeller, in order to adapt it to the case of a propeller moving with velocity v. These corrections consist in 1) substituting ϑ for ϑ' and r_0 for r'_0 from the relations (6a), thereby making allowance for aberration; and in 2) introducing the factors $\left(1-\frac{v}{c}\cos\vartheta'\right)^{-1}$ and $\left(1-\frac{v}{c}\cos\vartheta'\right)^{-2}$ dependent upon the torque and the force of traction, whereby allowance for the Doppler effect is made.

Theory proves that we are liable to depart widely from the actual state of affairs if the corrections we introduce on some consideration or other into the solution for a resting propeller are based on the elementary theory of aberration and Doppler effect. It seems natural, for instance, on passing from a resting to a moving propeller to substitute all frequencies ω with Doppler frequencies $\omega_1 = \omega\,(1 - v/c)\cos\vartheta'$, and yet it may be seen from (5) that under the sign of Bessel function we have an ordinary rather than a Doppler fre-

quency; the term involving the torque is multiplied by the Doppler factor $\left(1-\frac{v}{c}\cos\vartheta'\right)^{-1}$, though it does not contain ω, whereas the term dependent on the force of traction into which ω enters once as a factor, is multiplied twice by $\left(1-\frac{v}{c}\cos\vartheta'\right)^{-1}$.

If, further, the approximate methods developed by Gutin ([3]) are employed in computing the right-hand side of (5), we shall get a simplified formula as follows

$$B'_m = \frac{mn\omega}{2\pi c r'_0\left(1-\frac{v}{c}\cos\vartheta'\right)}\left(\frac{cM}{\omega R^2_{\text{mean}}}-\frac{P\cos\vartheta'}{1-\frac{v}{c}\cos\vartheta'}\right)J_{mn}\left(\frac{mn\omega}{c}R_{\text{mean}}\sin\vartheta'\right) \qquad (7)$$

Here P and M are force of traction and motor torque, respectively; R_{mean} is

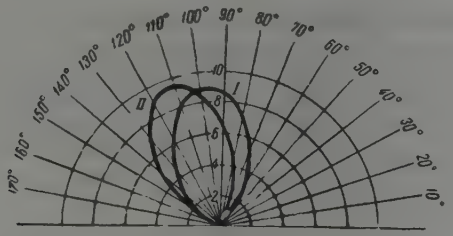

Fig. 1.

the mean value of the propeller radius R_0. It is customary to put $R_{\text{mean}} = 0.75\,R_0$.

For direct computation of B'_m as functions of r and ϑ the quantities r' and ϑ' in (7) should be substituted with r and ϑ by means of (6a). But the formula thus obtained is rather cumbersome. It is more convenient to carry out the calculation in two stages, first finding by means of (6a) the values of r' and ϑ' corresponding to the given values of r and ϑ, and then computing B' from (7). This method was applied by E. M. Fradkina in computing the first harmonic for a four-blade propeller with

$$R_0 = 3.6\text{ m},\ P = 608\ \text{kg},\ M = 754\ \text{kgm},\ \omega = 98.8\ \text{sec.}^{-1},\ r_0 = 50\text{ m}$$

The polar diagram represented here gives the amplitudes of sound pressure in bars (see figure). Curve *I* is for a propeller at rest; curve *II*, for a propeller moving at a rate of 360 km per hour. The diagram shows a considerable variation in the sound characteristic, which consists mainly in a backward shift and increase of the maximum. It will be obvious that in computing the intensity and direction of the sound radiated by the propeller the designer of modern aircraft must not neglect the influence of the forward motion.

At large distances from the propeller the sound field may, according to formula (5), be regarded as a field of a point source. Thus, for remote observation points the aberration of sound and the change in frequency due to the Doppler effect can be calculated from folmulae (6a) and (12a).

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30.I.1945.

REFERENCES

[1] M. D. Hart, Rep. and Mem. Aer. Res. Comm., No. 1310 (1929). [2] M. F. Shirokov, C. R. Acad. Sci. URSS, XLIX, No. 7 (1945). [3] Л. Я. Гутин, ЖТФ, VI, 899 (1936).

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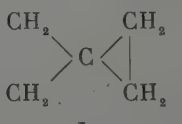
CHEMISTRY

SYNTHESIS OF SPIRO-(4,4)-NONANE

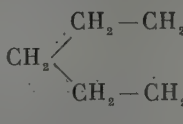
By N. D. ZELINSKY, Member of the Academy, and N. V. ELAGHINA

Hydrocarbons with a spiro-structure form a rather poorly investigated domain. Among the simplest spiro-hydrocarbons only spiro-(2,2)-pentane I (1), spiro-(4,5)-decane II (2) and spiro-(5,5)-undecane III (3) have been synthetized.

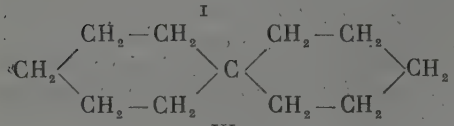
In order to extend our knowledge concerning the chemical and physical properties of the simplest spiro-hydrocarbons, the authors undertook the synthesis of spiro-(4,4)-nonane IV, a spiro-hydrocarbon the structure of which consists of two penta-methylene cycles.



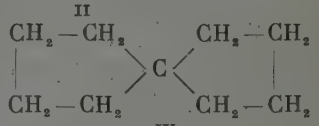
I



II

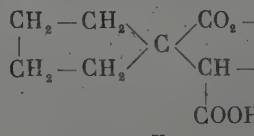


III



IV

In 1937 Chatterjee (4) made an attempt to prepare spiro-(4,4)-nonane IV starting from spiro-(4,4)-nonane-1-on-4-carboxylic acid V, synthetized in a very complicated way. The keto-acid V was reduced into spiro-(4,4)-nonane-1-carboxylic acid VI, and the calcium salt of the latter was subjected to thermal decomposition in the presence of lime.



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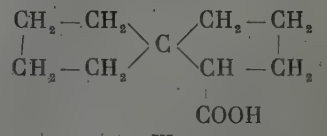
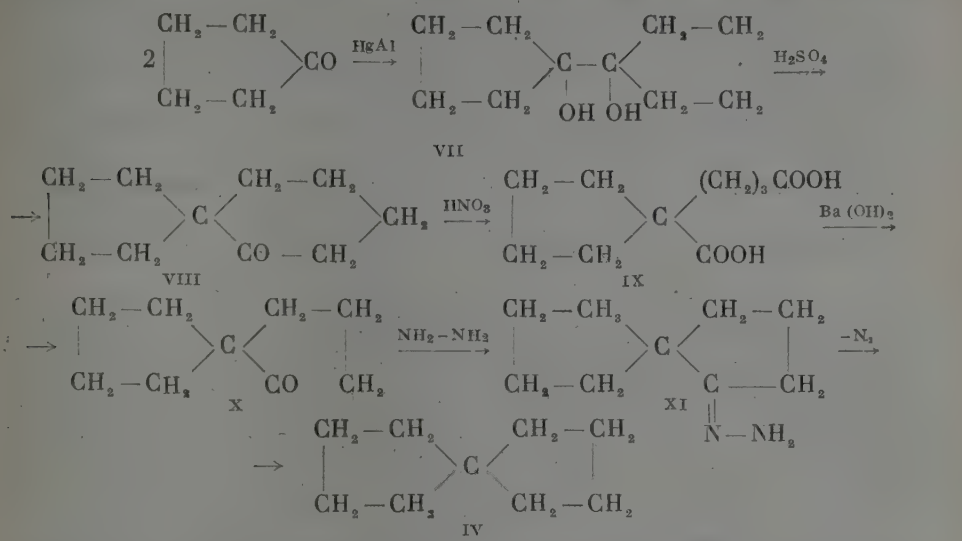


VI

However, the author was able to isolate from the reaction products only a very small amount of a fraction, b. p. 60°/12 mm, which, according to analytical data, corresponded to spiro-(4,4)-nonane.

In the present investigation spiro-(4,4)-nonane has been synthetized as follows. Cyclopentanone was reduced according to Barnett and Lawrence (5) with amalgamated aluminium into 1′.1-dihydroxy-1,1′-dicyclopentyl VII, which by heating with diluted sulphuric acid (1 : 1) was converted into spiro-(4,5)-decane-6-on VIII. The spiroketone VIII was oxidized with 50 per cent nitric acid in the presence of ammonium vanadate into cyclopentane-1-carboxy-1-butyric acid IX and the latter, when heated in the presence of barium hydroxide, was converted

into spiro-(4,4)-nonane-1-on X. The ketone X through the action of hydrazine-hydrate was converted into the hydrazone XI, and the latter was subjected to catalytic decomposition in the presence of potassium hydroxide and platinized charcoal.



Spiro-(4,4)-nonane IV is a colourless mobile liquid with an odour peculiar to terpenes and the following physical constants: b. p. 156.2—156.7°/748 mm; n_D^{20} = =1.4618; d_4^{20} =0.8631; MR_D =39.52 (calcul. for C_9H_{16} 39.36).

Below, for the case of comparison, are tabulated the properties of spiro-(4.4)-nonane obtained in the present work, and of spiro-(4,5)-decane obtained by Zelinsky and Shuikin (2).

Physical Properties of Spiro-(4,4)-nonane and Spiro-(4,5)-decane

	b. p.	d_4^{20}	n_D^{20}
Spiro-(4,4)-nonane	156.2—156.7°/748 mm	0.8631	1.4618
Spiro-(4,5)-decane	185—186°/745 mm	0.8877	1.4748

Experimental

1,1'-dihydroxy-1,1'-dicyclopentyl. In a 500 cm³ round-bottomed flask equipped with a reflux condenser and a dropping funnel were placed 20 g (0.74 g.-at.) of fine aluminium shavings and 100 cm³ of dry benzene. The benzene was brought to boiling on a water bath, and a solution of 10 g of sublimate in 100 g (1.19 moles) of cyclopentanone was added. The reaction mixture was refluxed on a boiling water bath with constant shaking for 2 hours. Then 85 cm³ of water and 150 cm³ of benzene were added and the mixture was refluxed for another 2 hours. The hot mixture was filtered with suction, and the precipitate was extracted with 150 cm³ of boiling benzene. The benzene solution was separated from the precipitate, added to the solution obtained previously, and benzene was distilled off until the temperature in the vapour phase reached 120—125°. The residue in the flask (a thick yellow oil) was cooled with ice. When petroleum ether was added, the pinacol separated out in the form of white crystals. The latter was filtered with suction, washed several times with petroleum ether and dried in the air.

Yield of 1,1'-dihydroxy-1,1'-dicyclopentyl 31.9 g (31.5 per cent of the theoretical); m. p. 108—109°.

S p i r o -(4,5)-d e c a n e -6-o n. In a 500 cm³ flask were placed 100 cm³ of concentrated sulphuric acid, 100 cm³ of water and 60 g (0.35 mol.) of 1,1′-dihydroxy-1,-1′-dicyclopentyl. The flask was connected to a reflux condenser, and the mixture was heated for 2 hours on a water bath. The spiro-ketone that was formed was washed with steam. The destillate was extracted with ether and the ether solution was washed with a 3 per cent sodium hydroxide solution and with water. After drying with anhydrous sodium sulphate the ether was distilled off and the residue distilled *in vacuo*.

Yield of spiro-(4,5)-decane-6-on 41.9 g (78.1 per cent of the theoretical). The substance had the following properties:

b. p. 99—100°/13 mm; 225—226°/743 mm; $n_D^{20}=1.4849$; $d_4^{20}=0.9890$: $MR_D=$ $=44.08$ (calcd. for $C_{10}H_{16}O$ 43.99)

Zelinsky and Shuikin (²) give the following constants for spiro-decanone:

b. p. 98—100°/13 mm; $n_D^{20}=1.4848$; $d_4^{20}=0.9898$.

C y c l o p e n t a n e -1-c a r b o x y -1-b u t y r i c a c i d. In a 250 cm³ flask connected to a reflux condenser and a dropping funnel were placed 90 cm³ (0.93 mol.) of nitric acid (sp. weight 1.4), 40 cm³ of water and 0.1 g of ammonium vanadate. The flask was heated on a water bath to 55—60° and that temperature was maintained to the end of the reaction. Gradually, with constant shaking from the dropping funnel 30 g (0.2 mol.) of spiro-(4,5)-decane-6-on were added. Finally, the reaction mixture was heated for 1 hour on a boiling water bath. The solution was filtered and evaporated on a water bath. The sirupy residue crystallized after cooling. The acid was filtered off, washed with cold water and dried in a dessiccator over sulphuric acid.

Yield of cyclopentane-1-carboxy-1-butyric acid was 26.8 g (67.9 per cent of the theoretical); m. p. 91—92° (after crystallization from a mixture of acetone and petroleum ether).

S p i r o -(4,4)-n o n a n e -1-o n. A 100 cm³ Würtz flask connected to a condenser and provided with a thermometer, whose bulb was 3 mm away from the bottom of the flask, received a finely ground mixture of 20 g (0.1 mol.) of cyclopentane-1-carboxy-1-butyric acid and 1 g of crystalline barium hydroxide. The flask was heated on a metallic bath, the temperature of which was raised gradually. The spiroketone was distilled off when the temperature of the reaction mixture was 295—300°. The distillate was saturated with solid potassium carbonate, and the ketone was separated, dried with melted potassium carbonate and distilled.

Yield of spiro-(4.4)-nonane-1-on was 12.4 g (89.6 per cent of the theoretical). The substance had the following properties:

b. p. 202—203°/760 mm; $n_D^{20}=1.4770$; $d_4^{20}=0.9842$; $MR_D=39.65$ (calcd. for $C_9H_{14}O$ 39.37)

S p i r o (4,4)-n o n a n e. In a 150 cm³ flask were placed 34 g (0.24 mol.) of spiro-(4,4)-nonane-1-on, 27.3 g (0.49 mol.) of 90 per cent hydrasine-hydrate and 42 cm³ of ethyl alcohol. The flask was connected to a reflux condenser and its content was heated for 6 hours on an oil bath, the temperature of which was maintained at 120°. The alcohol and the remaining hydrazine-hydrate were distilled off, the temperature of the oil bath being raised towards the end of the distillation to 140°. The residue in the flask was saturated with solid potassium hydroxide, and the upper layer, consisting of the hydrazone, was separated from the aqueous one.

The hydrazone was dried with freshly melted potassium hydroxide at first in the cold and then under heating to 100° on an oil bath. Yield of the hydrazone of spiro-(4,4)-nonane-1-on was 35.4 g. The hydrazone, a thick yellow oil, was subjected to catalytic decomposition.

In a 100 cm³ Würtz flask fitted out with a thermometer and a condenser were placed 35.4 g of the hydrazone of spiro-(4,4)-nonane-1-on and 2 g of freshly fused potassium hydroxide ground into a fine powder and a small lump of platinized charcoal. The mixture was heated on a metallic bath, the temperature of which was

raised gradually. When the temperature of the bath reached 160°, a vigorous decomposition of the hydrazone took place.

The main amount of the hydrocarbon distilled off when the temperature of the bath was 185—190° and that of the distilling vapours 153—155°. Towards the end of the experiment the temperature of the bath was raised to 200°. The distillate was distilled repeatedly over a fresh catalyst.

The product obtained was washed with 50 per cent acetic acid, with water, with a 3 per cent potassium hydroxide solution, again with water and dried with fused potassium carbonate. After fractional distillation over metallic sodium 21 g of spiro-(4,4)-nonane were obtained (68.7 per cent of the theoretical), having the following constants:

b. p. 156.2 — 156.7°/748 mm; $n_D^{20} = 1.4618$; $d_4^{20} = 0.8631$; $MR_D = 39.52$
(calcd. for C_9H_{16} 39.36).

Analysis I: Substance 6.670 mg; 7.681 mg H_2O; 21.332 mg CO_2
» II: » 5.002 mg; 5.637 mg H_2O; 16.000 mg CO_2
C_9H_{16} Found I,%: C 87.22; H 12.88
II,%: C 87.24; H 12.61
Calc,%: C 87.01; H 12. 99.

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Received
9. X. 1945.

REFERENCES

[1] Н. Д. Зелинский и В. П. Кравец, ЖРФХО, **44**, 1873 (1912); Ber., **46**, 160 (1913); F. Rogowski, Ber., **72**, 2021 (1939); M. I. Murray and E. H. Stevensen, J. Am. Chem. Soc., **66**, 314, 812 (1944); J. Donahue, G. L. Humphrey and V. Schomaker, J. Am. Chem. Soc., **67**, 332 (1945). [2] Н. Д. Зелинский и И. И. Шуйкин, ЖРФХО, **61**, 2245 (1929); Ber., **62**, 2180 (1929). [3] W. S. G. P. Norris, J. Chem. Soc., **1926**, 245. [4] N. N. Chatterjee, J. Indian Chem. Soc., **14**, 259 (1937). [5] E. de Barry Barnett and C. A. Lawrence, J. Chem. Soc., **1935**, 1104.

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CHIMIE

CRISTALLISATION ISOTHERMIQUE (35°) DES SELS LORS DE L'ÉVAPORATION DES EAUX DE LA MER D'ARAL

Par V. I. NIKOLAEV et KH. B. FRADKINA

(Présenté par I. I. Černjaiev, de l'Académie, le 13. XII. 1944)

L'air extrêmement sec et la température bien élevée en été (jusqu'à 50—60°), en printemps et même en automne ont été les principaux agents naturels, favorisant la formation des lacs sulfatés, dits «sulfatniki», dans les régions de la mer d'Aral (lac Djaksy-Klytch et autres). Afin d'élucider leur genèse et d'établir le mode de leur exploitation rationnelle en industrie nous avons examiné les allures de cristallisation des sels qui se dégagent à l'évaporation des eaux de la mer d'Aral, qui baignait autrefois de ses ondes les régions des lacs sulfatés.

La figure donne le diagramme de cristallization des sels à l'évaporation isothermique (35°) des eaux de cette mer. Les nombres des molécules $CaSO_4$, $MgSO_4$, $MgCl_2$, ainsi que ceux de la somme Na_2Cl_2 et K_2Cl_2 y sont portés dans quatre directions. Ces grandeurs sont calculées par rapport à mille molécules de H_2O.

Les allures de cristallisation des sels sont composées des segments suivants:

Segment *ab* — cristallisation du gypse $CaSO_4 \cdot 2H_2O$
» *bc* — » » $NaCl$
» *cd* — » » $NaCl + MgSO_4 \cdot Na_2SO_4 \cdot H_2O$ (astrakhanite)
» *de* — » » $NaCl + MgSO_4 \cdot 7H_2O$ (epsomite)
» *ef* — » » $NaCl + MgSO_4 \cdot 6H_2O$
» *feu* — » » $NaCl + MgSO_4 \cdot 6H_2O + KCl \cdot MgSO_4 \cdot 3H_2O$ (kaïnite)

Le processus aboutit au point eutonique, où la cristallisation des sels suivants a lieu: $MgCl_2 \cdot 6H_2O$ (bischofite), $KCl \cdot MgSO_4 \cdot 3H_2O$ (kaïnite), $NaCl$ et $MgSO_4 \cdot 5H_2O$ (pentahydrate de sulfate de magnesium).

De cette façon l'eutonie métastable du pentahydrate fut démontrée et déterminée au moyen de l'analyse chimique ainsi que par la méthode cristallo-optique.

Ce phénomène est dû à la température et aux coefficients élevés de la métamorphisation des eaux-mères (pour la mer d'Aral $K = \frac{MgSO_4}{MgCl_2} = 38$). La concentration de $MgSO_4$, égale à 3.33 pour cent, et celle de $MgCl_2$—33.88 pour cent (à 35°), caractérisent le composé de cette eutonie.

L'analyse de chaque segment de la courbe de cristallisation nous conduit aux conclusions suivantes:

1. La cristallisation du gypse s'opère le long du segment *ab* comme suit: on voit l'accumulation de $CaSO_4$ jusqu'au point *m* (jusqu'à 0.51 pour cent), suivie d'un dégagement rapide et abondant de $CaSO_4 \cdot 2H_2O$ formant la phase du fond. En pratique les eaux-mères ne contiennent pas de $CaSO_4$ au point *b*.

Etant donné que le long de *ab* les eaux-mères passent une phase de concentration, pendant laquelle le refroidissement provoque la cristallisation de mirabilite $Na_2SO_4 \cdot 10H_2O$, celle-ci est considérablement souillée par le gypse.

Ainsi la cause principale du souillage de la mirabilite et du sulfate des lacs sulfatés d'Aral par le gypse (jusqu'à 10—15 pour cent) est trouvée. Le sulfate purifié, de haute qualité, est obtenu au moyen de la cristallisation successive de la mirabilite et du sulfate par dissolution à l'eau, suivie de dessiccation favorisée par les conditions climatiques de cette région.

Pendant l'expédition d'été de 1944 V. I. Nikolaev et E. N. Tverdova ont fait une tentation de dissoudre à l'eau la mirabilite souillée, et d'évaporer la solution dans les conditions climatiques des mines de sulfate dans la région de la mer d'Aral. Il fut constaté qu'au bout de 5—6 jours il ne restait plus d'eau, dans laquelle la mirabilite était dissoute (solution de Na_2SO_4 de 22 pour cent), et ensuite au bout d'un mois toute la mirabilite, cristallisée successivement sur la surface du gypse et des impuretés entraînées mécaniquement, était transformée en sulfate par déhydration. Le sulfate ainsi obtenue ne contenait que très peu d'impuretés (sable, argile, *etc.*) et même de gypse. La teneur en $CaSO_4$ ne dépassait pas 0.5 pour cent.

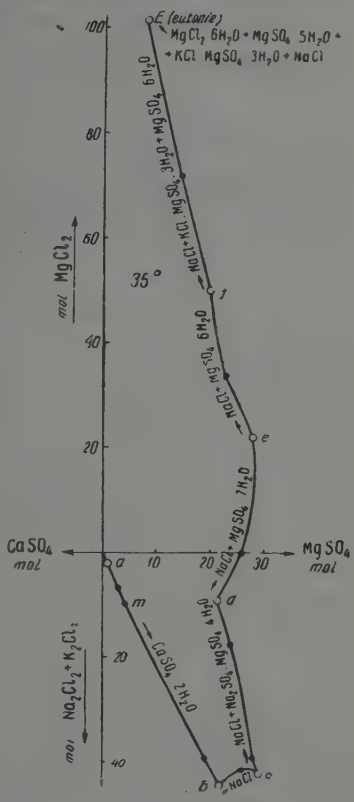

Fig. 1

2. Le chlorure de sodium est cristallisé le long du segment court *bc*. Cette cristallisation avait déjà eu lieu dans la vallée du lac Djaksy-Klytch, dans laquelle se rassemblaient les eaux-mères après le dégagement préalable de la mirabilite dans les nombreux lacs sulfatés et après avoir atteint la concentration de dégagement de NaCl.

3. La cristallisation de l'astrakhanite s'ajoute à celle de NaCl le long du segment *cd*. C'est ici que la cristallisation primaire de ce minéral a été observée pour la première fois. Un coefficient élevé de transformation, égal à 38 au début de la cristallisation (au point *c*), autrement dit, une teneur insignificante en $MgCl_2$ dans les eaux-mères, constitue une condition favorable à ce phénomène.

L'expérience prouve (1) que plus la concentration de $MgCl_2$ en solution est grande (autrement dit, plus le coefficient de transformation est petit), plus la formation d'astrakhanite est lente (atteignant des vitesses infiniment petites). Nikolaev et ses collaborateurs (2) ont élaboré les méthodes de transformer ce minéral en mirabilite-sulfate.

4. La cristallisation de l'epsomite est notée le long des segments *de* et *ef* de notre diagramme. Le long de ces segments la concentration de $MgCl_2$ en solution atteint des valeurs telles (10—20 pour cent) que la formation d'astrakhanite cesse.

La présence d'epsomite au-dessus des gisements de l'astrakhanite est confirmée au moyen des fouilles faites dans le lac de Djaksy-Klytch (4). Le sulfate de magnésium tel quel peut avoir une signification en industrie, ou il peut être employé avec NaCl pour obtenir la mirabilite.

5. La cristallisation de la kaïnite a lieu le long du dernier segment *feu* de notre diagramme.

La concentration des eaux-mères du lac Djaksy-Klytch baignant les gisements salins de NaCl, de l'astrakhanite et de l'epsomite ne présentent pas les hautes valeurs de concentration qui correspondrait au segment *feu*. Mais en effectuant l'évaporation dans les bassins artificiels (marais salins) on peut parvenir à la cristallisation de la kaïnite et même du chlorure de magnésium «solaire» dans les conditions climatiques de la région de la mer d'Aral.

Au point de vue théorique il ne serait pas dénué d'intérêt de souligner que la teneur élevée en $MgSO_4$ dans les eaux-mères et l'augmentation de la température jusqu'à 35° empêchent la formation de carnallite métastable, comme cela a lieu à 25° pour les eaux-mères de la Mer Noire, dont le coefficient de transformation est petit (5).

La kaïnite ainsi que l'astrakhanite présentent des phases solides primaires prouvées par l'analyse des allures de cristallisation.

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LITTÉRATURE CITÉE

[1] В. И. Николаев и А. И. Качалов, Усп. химии, IX, 6 (1940). [2] Г. С. Клебанов, Д. М. Корф и Л. В. Еловская, Сборн. «Приаральский соляной район», изд. АН СССР, 1937. [3] В. И. Николаев и Л. С. Дынкина, ЖПХ, 9, в. 12 (1936); V. I. Nikolaev et E. N. Tverdova, C. R. Acad. Sci. URSS, XLVI, No. 8 (1945). [4] Ф. Ф. Бадер, Журн. «Исследования озер СССР», в. 6 (1934). [5] Н. С. Курнаков и В. И. Николаев, Изв. Ин-та физ.-хим. анал., 10 (1938).

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PHYSICAL CHEMISTRY

ON THE DIELECTRIC PROPERTY AND ELECTROSTRICTION OF SOLUTIONS OF ELECTROLYTES

By O. K. DAVTJAN

(Communicated by G. M. Krzhizhanovsky, Member of the Academy, 28. XII. 1944)

It is known that a polar dielectric, when acted upon by a strong electric field, simultaneously shows electrostriction, or compression, and a decrease in the dielectric constant, *i. e.* an electric saturation effect. It is also known that both these phenomena are observed in solutions of electrolytes as well. These phenomena are due to the combined action of the electric fields of the ions. Each individual ion in the solution sets up around it an electric field of great strength. Herewith, orientation of the dipole molecules and, hence, a decrease in the polarization of the liquid, should take place in the solution, on the one hand, and extension of the dipoles to the ions, which leads to a decrease in the volume of the liquid, *i. e.* to electrostriction, on the other.

It may be shown that these phenomena are connected by a definite functional relationship. For the mean electrical moment $\overline{m}$ of a molecule in a field F Debye arrived at the following expression:

$$\overline{m} = \mu L(x) + \alpha_0 F, \quad x = \mu F/kT \tag{1}$$

where $L(x) = \operatorname{cth} x - 1/x$ is the Langevin function, μ is the electrical moment of a single molecule, F is the strength of the «internal» field or of the effective electric field (in the given case, of the ions), and α_0 is the optical polarizability of the molecules or the deformational polarization. Expanding $\operatorname{cth} x$ in series, we may express the mean electric moment thus:

$$\overline{m} = \mu \left(\frac{x}{3} - \frac{x^3}{45} + \ldots \right) + \alpha_0 F \tag{2}$$

On the other hand, Debye [1] on the basis of thermodynamics derived the formula

$$\Delta v_{p,T} = -v \left(\frac{\partial \varkappa}{\partial p} \right)_T \frac{E^2}{2} \tag{3}$$

where $\Delta V_{p,T}$ is the change in specific volume under the action of an electric field at constant pressure and constant temperature; $\varkappa$ is the electric susceptibility of the solvent; p is the pressure; E is the electric field strength and T is the absolute temperature.

In deriving formula (3) the condition that an external field act on the liquid was imposed. However, we may assume that to a given reduction in volume and in the mean electric moment under the action of the electric field of the ions there corresponds a definite value of the external electric field, say, of the field of a flat condenser.

Proceeding from equation (2) we may obtain the dependence of the change in the dielectric constant upon the field strength. If the saturation effect is not

taken into account, which is permissible only when the electric field strength is sufficiently small, we may confine ourselves to the first approximation of the Langevin function. Then the following expression is obtained for the dielectric constant of the liquid:

$$\varepsilon_0 = \frac{F + 4\pi I_0}{E} \qquad (4)$$

where $I_0 = n_0 (\mu^2/3\,kT + \alpha_0) F$ is the electric moment of a unit volume and k is the Boltzman constant. In equation (4) the dielectric constant is larger than the actual value, for the saturation effect is not taken into account. When this effect is taken into account, it is necessary to retain the second term, proportional to the third power of x. Then

$$\varepsilon_0 = \frac{E + 4\pi I_1}{E};\quad I_1 = n_1 \left(\frac{\mu^2}{3kT} F - \frac{\mu^2}{45\,(kT)^3} F^3 + \alpha_0 F \right) \qquad (5)$$

Here n_1 is the number of molecules in 1 cm³ after isothermal compression.

From equations (4) and (5) we get the decrease in the dielectric constant

$$\Delta\varepsilon = 4\pi \left(\frac{\mu^2}{3kT} + \alpha_0 \right) (n_0 - n_1)_{p,T} \frac{F}{E} + 4\pi n_1 \frac{\mu^4}{45\,(kT)^3} \frac{F^3}{E} \qquad (6)$$

It is found that in equation (6) F may be expressed through E without having to look for a method of determining the effective field of the dipole molecules. As a matter of fact, the force F_0 acting in the solution on one of the molecules may be resolved into three components: $F_0 = F_1 + F_2 + F_3$, where $F_1 = 4\pi\sigma$ and $F_2 = -4\pi I + \frac{4\pi'}{3} I$ (σ being the surface density of the charges on the armature of the condenser). As for F_3, it is due exclusively to the forces of interaction of the polar molecules of the liquid.

Thus, the effective field F of the ions consists only of the first two components, *i. e.* $F = F_1 + F_2 = F_0 - F_3$, or $F = E + 4\pi I$. Keeping in mind further that $D = E + 4\pi I$, we get $F = \frac{\varepsilon_0 + 2}{3} E$. Substituting the last expression into equation (6), we get

$$\Delta\varepsilon = \frac{4\pi}{3} \left(\alpha_0 + \frac{\mu^2}{3kT} \right) (n_0 - n_1)_{p,T} (\varepsilon_0 + 2) + 4\pi n_1 \frac{\mu^4}{45\,(kT)^3} \left(\frac{\varepsilon_0 + 2}{3} \right)^3 E^2 \qquad (7)$$

Further, eliminating E^2 from equations (3) and (7), and neglecting the small quantities, we get the dependence between the change in the dielectric constant and the electrostriction

$$\Delta\varepsilon = -D_T \rho_1 \frac{\Delta v_{p,T}}{v_1} \qquad (8)$$

where $D = 0.712\ \pi^2 N_A \frac{\mu^4}{M\,(kT)^2} \frac{1}{(\partial\varepsilon/\partial p)_{p,T}} \left(\frac{\varepsilon_0 + 2}{3} \right)^3$ is a constant for a given solvent at constant temperature, N_A being the Avogadro number. For water $\varepsilon_0 = 81$, $\mu = 1.7 \cdot 10^{-18}$ and $(\partial\varepsilon/\partial p)_{p,T} = 4.6 \cdot 10^{-3}$ 1/kg (²); then at 18°C $D_{18} = 1.38 \cdot 10^5$.

Thus, we have established that the change in the dielectric constant of a polar liquid consequent upon the dissolution of an electrolyte is proportional to its electrostriction.

Equation (8) gives only the difference between the dielectric constant of the pure solvent and of the solvent after the introduction of ions into it. However, calculation shows that $\Delta\varepsilon$ in equation (8) is also the difference between the dielectric constants of the solution and the solvent. Heydweiller (³) has proposed the formula

$$\Delta = \left[\frac{\lambda}{\lambda_0} (A - B) + B \right] c \qquad (9)$$

where Δ is the per cent change in density, c is the concentration in gram-equivalents per litre, A and B are constants giving respectively the per cent

change in density caused by one gram-equivalent of the ionized and unionized electrolyte, and λ and λ_0 are the equivalent small concentrations, respectively. Since it may be assumed that for dilute solutions $\lambda/\lambda_0 = 1$, then for the absolute change in density, caused only by the ionization of the electrolyte, we get the expression

$$\frac{\Delta v_{p,T}}{v_1} = \frac{(v_1 - v_0)_{p,T}}{v_1} = -(A-B)\, c \cdot 12^{-2} \qquad (10)$$

Hence if (9) is taken into consideration, the formula expressing the relation between the change in the dielectric constant and the concentration of the electrolyte will have the form

$$\varepsilon_{1,2} = \varepsilon_0 \left(1 - \frac{D\rho_0 (A-B)\, 10^{-2}}{\varepsilon_0} c\right) \qquad (11)$$

or

$$\varepsilon_{1,2} = \varepsilon_0 (1 - \gamma c) \qquad (12)$$

where $\gamma = D\rho_0 (A-B)\, 10^{-2}/\varepsilon_0$ is a constant for a given solution.

Salt	$A-B$ at 18 °C	γ	γ_1	γ_2	γ_3
LiCl . .	0.53	7.55	7.4	—	16.0
KCl . .	2.2 (1.82 at 30 °C)	36.5 (25.0 at 30 °C)	6.9	20.2	—
NaCl . .	1.02	17.5	7.1	20.4	21.0
$BaCl_2$.	1.06	22	8.3	22.5	—
KOH . .	1.54	26.5	—	26.5	—
NaOH .	1.51	26.0	—	—	32

Since for water at 18°C $D = 1.38 \cdot 10^5$ and $\rho = 0.998$, then $\gamma = 17.2\ (A-B)$.

It is interesting that the researches of Sack([4]), as well as of other investigators, showed that for solutions of small concentration the dielectric constant decreases with an increase in concentration and may be expressed by an equation of the form of (12).

The accompanying table gives $A-B$, the values of γ calculated with formula (12), and also the values of γ obtained experimentally by Sack (γ_1), Walden (γ_2) and Penhold (γ_3) for some salts. It must be kept in mind, however, that the accuracy of the experimental data is highly questionable since it is very difficult in principle to determine the dielectric constant of a solution. The table shows that, notwithstanding the approximate nature of the experimental data, their agreement on the whole with the theoretical values of γ is satisfactory.

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REFERENCES

[1] P. Debye, Handb. d. Radiologie, VI, 750, 1925. [2] Техническая энциклопедия, Справ. физич. химии и технич. величин, 4. [3] A. Heydweiller, Ann. Phys., 30, 873 (1909). [4] H. Sack, Phys. Z., 28, 199 (1927); P. Debye, Polar molecules, 1931.

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GEOCHEMISTRY

ON SCANDIUM CONTENT IN CHLOPINITE

By S. A. BOROVICK

(Communicated by D. S. Beliankin, Member of the Academy, 26. X. 1945)

Scandium is one of the most widely disseminated elements. Quantitative determinations carried out under application of a most elaborate technique have shown that out of the 257 specimens of various minerals and rocks obtained from the East Indies, three specimens only proved devoid of scandium, one specimen was found to contain 0.02 per cent, in 34 specimens the element was detected in thousandths fraction of a per cent, and the remaining 219 were found to hold $n \cdot 10^{-4}$ per cent of scandium [1].

A similar picture may be reconstructed after data on spectrum analyses, adduced by various writers [2,3,4]. Of particular interest are therefore products showing an increased scandium concentration.

Aside from the scandium mineral, tortweightite (43 per cent of Sc_2O_3), viikite, bacite and orthite have likewise been found to contain increased quantities of scandium. All these minerals are of very rare occurrence, however.

The present note contains data on the content of scandium in chlopinite. This mineral which owes its name to V. G. Chlopin, member of the Academy, is described by the formula $FeYtU^4(TiO_2)_3(NbO_4)_5(OH)$; until now no data as to its scandium content have been available.

The very first spectrograms of chlopinite taken for qualitative analysis showed very intense scandium lines. On account of the unusually high concentration of the element we were interested in, the original product had to be mixed with feldspar which is known to contain no scandium, first at a ratio 1 : 4, and further on, 1 : 8.

Three standard mixtures were also prepared with feldspar, the content of Sc_2O_3 being of 1, 0.2 and 0.066 per cent, respectively. By applying the method of the three standards, the measurement error could be reduced to ± 20 per cent.

In addition to chlopinite from the Verkhne-Udinsk deposits, analysed was also a sample of chlopinite from Baschu-Kunduy. This mineral was found to contain about the same quantity of Sc_2O_3 as the former.

Specimen	Place of sampling	Content in %
Chlopinite	Baschu-Kunduy	1.1
»	Verkhne-Udinsk	0.9
Feldspar	»	0.01
Biotite	Baschu-Kunduy	0.01
Grey film	»	No scandium detected

Moreover, chlopinite, enclosing feldspar biotite from the same piece of ore, as well as the light-grey film which formed intercalations between the laminae of this biotite, were also found to contain scandium. The corresponding data are to be found in the table.

Thus, a raw material exceedingly high in scandium has been discovered in the USSR.

This should be taken into account when treating chlopinite; and necessary measures are to be taken in order to preserve and utilize the waste products which may contain industrial accumulations of scandium.

In concluding I wish to express my thanks to K. A. Nenadkevich and A. N. Labunzov for having kindly put at my disposal the specimens of chlopinite referred to above.

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REFERENCES

[1] W. Tongeren, On the Occurrence of Rare Elements in the Netherland East Indies. Amsterdam, 1938. [2] G. Ebergard, Sitzungsber. Akad. d. Wissensch. Berlin (1908), [3] V. M. Goldschmidt u. Cl. Peters, Nachr. Ges. Wissensch. Göttingen (1931). [4] S. A. Borovick, C. R. Acad. Sci. URSS, XL, No. 3 (1943).

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CHIMIE BIOLOGIQUE

DIFFUSION ET POIDS MOLÉCULAIRE DE LA STURINE

Par V. PLASKÉEV, N. YAROVENKO et A. PASSYNSKI

(Présenté par A. N. Bach, de l'Académie, le 8. XII. 1944)

La sturine—protamine contenue dans le sperme de l'esturgeon, fait partie avec tout le groupe des protamines des albumens de structure particulièrement simple, dont nous avons la possibilité d'établir par la voie chimique non seulement la structure élémentaire, mais également de procéder à l'étude complète de leurs acides aminés. Cependant, il est bien connu que l'analyse chimique ne nous donne que la valeur minime du poids moléculaire, différent du poids moléculaire réel d'un certain coefficient entier. En effet, cette méthode a permis d'établir pour la sturine-sulfate des valeurs du poids moléculaire minime variant entre 800 et 3500 (voir plus loin).

Etant donné que la valeur du poids moléculaire a une importance essentielle pour l'établissement de la structure d'un corps donné, nous nous sommes attachés dans le présent travail à déterminer le poids moléculaire de la sturine (obtenue à partir du sperme de l'esturgeon russe—*Acipenser güldenstädti*) par la méthode de Lamm-Polson ([1],[2]) en mesurant la diffusion de la sturine.

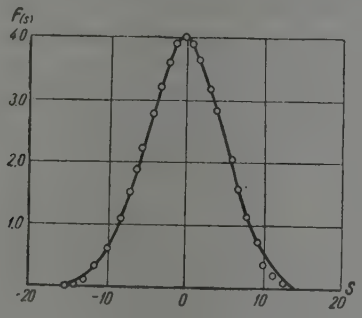
La sturine-sulfate était obtenue à partir de sa solution dans l'acétone par précipitation par une solution à 20 pour cent de H_2SO_4, après précipitation préalable de la sturine par l'acide acétique à partir du sperme de l'esturgeon et la séparation des acides nucléiques ([3]). La sturine-sulfate obtenue avait la constitution élémentaire suivante: teneur en eau—9.05 pour cent, N=22.37 pour cent, C=37.44 pour cent, H=6.89 pour cent, SO_4H_2=22.45 pour cent. L'analyse des acides aminés a permis de mettre en évidence la présence de 20 acides aminés ([3]). La sturine-sulfate obtenue se dissolvait dans Na_2SO_4 à 1 pour cent; cette solution d'une concentration de 0.2 pour cent de sturine-sulfate dans Na_2SO_4 à 1 pour cent nous a servi de solution-type dans nos expériences pour mesurer la diffusion et la viscosité; nous nous sommes servis d'une solution de Na_2SO_4 à 1 pour cent comme couche liquide supérieure dans nos expériences sur la diffusion. Toutes ces déterminations furent effectuées à la température de 20° C$\pm$0.01. La méthode employée pour ces déterminations et les calculs est décrite ailleurs ([1],[2]).

La valeur du coefficient de diffusion de la sturine-sulfate dans Na_2SO_4 à 1 pour cent rapportée à l'eau pure constitue $1.5—1.75 \cdot 10^{-6}$ cm^2/sec., en moyenne $D_{20^0,w} = 1.62 \cdot 10^{-6}$ cm^2/sec.; la durée de notre expérience sur la diffusion était de deux heures. Les données expérimentales normalisées répondent bien à la courbe théorique de Gauss (voir la figure), ce qui nous confirme l'homogénéité moléculaire du produit et l'absence d'anomalies dues à la charge électrique des particules ou à la concentration de la sturine-sulfate. La constante de viscosité des solutions diluées de la sturine-sulfate dans Na_2SO_4 à 1 pour cent est égale à $k = \frac{\eta_r^{-1}}{vc} = 8.25$ (η_r, la viscosité relative de la solution; v=0.75, volume partial; et c, concentration de la sturine-sulfate dans un g/cm^3). D'après l'équation de Polson ([4]) $k = 4.0 + 0.098\ (b/a)^2$,

d'où si $k=8.25$, $b/a=6.6$; à cette valeur de b/a correspond le coefficient de dissymétrie [2] $f/f_0=1.34$. Les données précitées nous permettent de calculer le poids moléculaire M d'après la formule:

$$M=\frac{k'}{(f/f_0)^3 D^3 v}$$

où $k'_{20^\circ,w}=2.41\cdot10^{-14}$; en faisant $f/f_0=1.34$, $D=1.62\cdot10^{-6}$ et $v=0.75$, nous avons $M=3100$. Ainsi donc, la sturine-sulfate d'après nos déterminations a un poids



moléculaire $M=3100$ et est constituée de particules semblables, de forme allongée, avec une relation de leurs demi-axes $b/a=6.6$.

Il est intéressant de comparer à cette valeur de M les résultats des analyses chimiques de la sturine-sulfate de l'esturgeon russe, que l'on rencontre dans la littérature. Dans le tableau nous indiquons le nom de l'auteur, la formule empirique de la sturine-sulfate proposée, le poids moléculaire minime calculé d'après cette formule, le coefficient entier dont l'emploi rapproche le plus cette valeur du poids moléculaire $M=3100$ que nous avons trouvé, et enfin, les valeurs du poids moléculaire réel de la sturine-sulfate.

Auteur	Formule empirique	Poids moléculaire minime	Coefficient entier	Poids moléculaire réel
V. Malenük [5]	$C_{37}H_{55}N_{13}O_7\cdot2H_2SO_4$	869	4	3476
M. Lisitsyn et A. Alexandrovskaia [6]	$C_{35}H_{74}N_{18}O_7\cdot3H_2SO_4$	1152	3	3456
N. Yarovenko [3]	$C_{108}H_{203}N_{57}O_{23}\cdot8H_2SO_4$	3451	1	3451

Ainsi, les valeurs du poids moléculaire de la sturine-sulfate calculées d'après les données chimiques que différents auteurs ont obtenues indépendamment l'un de l'autre en des temps différents, s'accordent remarquablement entre elles. Se basant sur les données du tableau on a des raisons suffisantes de prendre comme poids moléculaire de la sturine-sulfate la valeur de $M=3460$; la valeur expérimentale déterminée par nous $M=3100$ n'en diffère que de 10 pour cent, ce qui est parfaitement acceptable. La molécule de sturine-sulfate contenant 8 H_2SO_4 (voir le tableau), il s'ensuit que pour la sturine pure nous avons $M=2680$ ou, en arrondissant, $M=2700$.

Ayant une relation entre les demi-axes des particules de $b/a=6.6$ et une densité de $D=1.3$, le poids moléculaire de $M=2700$ correspond, comme l'indique un simple calcul, à des particules ayant une forme d'ellipsoïde allongée, d'une longueur de 64 Å et d'une largeur de 9.8 Å; comme il est indiqué plus haut, une telle particule contient 20 acides aminés. Sans attacher une valeur absolue à ces données il importe d'indiquer tout de même que ces chiffres sont tout à fait satisfaisants,

soit qu'on considère la longueur d'un acide aminé dans une chaîne (3.2 Å), soit la longueur de la chaîne latérale (4.5—5.0 Å). Ainsi la molécule de la sturine a vraisemblablement la forme d'une simple chaînette polypeptique.

Enfin, il importe d'indiquer qu'à notre avis notre conclusion sur l'homogénéité de structure des particules de la sturine se trouve confirmée par les travaux de Kossel et Schenk (7), qui ont réussi à obtenir la fractionisation de la sturine contrairement aux autres protamines (saprinine, cloupéine), étudiées par eux; ces auteurs en concluent à l'homogénéité chimique de la sturine.

Ainsi donc, on peut considérer que l'étude de la diffusion et de la viscosité des solutions d'albumen par la méthode de Lamm-Polson permettent d'obtenir des résultats parfaitement valables, caractérisant les propriétés moléculaires des corps semblables à la sturine. Cette conclusion est importante, car cette méthode a été vérifiée à son début par Polson (4) par comparaison avec les données obtenues par ultra-centrifugation des albumens à partir de $M = 30\,000$ seulement (pour M compris entre 30 000 et 6 500 000). Notre travail sur la détermination du poids moléculaire du lysocime (8) a déjà montré que la méthode de diffusion donne des résultats correspondant bien aux données chimiques pour les corps de $M = 12\,000$—$13\,000$; le présent travail montre une bonne correspondance des résultats des deux méthodes pour des valeurs de M de l'ordre de 3000. Ainsi donc, les résultats obtenus permettent d'élargir considérablement l'application de la méthode donnée d'étude des propriétés moléculaires des albumens, au domaine intéressant des solutions de protéines à petites molécules et des produits de destruction de l'albumine.

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[1] O. Lamm and A. Polson, Biochem. J., **30**, 538 (1936). [2] А. Пасынский, Усп. химии, **10**, 526 (1941). [3] Н. Яровенко, Диссертация. ВИЭМ, 1944. [4] A. Polson, Koll. Z., **87**, 149 (1939). [5] W. Malenük, Z. physiol. Chem., **57**, 99 (1908). [6] М. Лисицын и Н. Александровская, Тр. лаборатории по изучению белка. № 1, 27 (1940). [7] A. Kossel u. E. Schenk, Z. physiol. Chem., **173**, 293 (1928). [8] A. Passynski et V. Plaskéev, C. R. Acad. Sci. URSS, XLVII, No. 8 (1945).

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GEOLOGY

ON THE PALAEOGENE OF THE NAUKAT DEPRESSION

By O. S. VIALOV

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In spite of the fact that the Palaeogene deposits of Ferghana are supposed to be rather thoroughly investigated, there still remain certain regions about the Palaeogene of which insufficient or even incorrect information is available. Among such regions should be classed the Naukat depression, situated in South Ferghana (between the meridian of the town of Osh and the town of Ferghana, south of the Katran Palaeogene massif). Very brief references to the Tertiary deposits of the Naukat depression are found in papers by V. N. Weber, I. S. Komishan and others. The geologists named above were not specially engaged in the study of Palaeogene deposits, and at that time the stratigraphy of these deposits in Ferghana had not been worked out. The information referred to by these authors fails to give any idea as to the nature of the Palaeogene. More interesting data are contained in a paper of V. I. Solun who, however, erroneously points to the absence of the Upper Ferghana subdivision in the section and does not record the faunistically characterized Bukhara stage.

The first finding of fauna of the Upper Ferghana subdivison (Sumsar stage) was made by O.A. Rischkov who gave a more correct interpretation of the section in his manuscript kept in the archives of the «Kalininneft» Trust.

In 1943 the author visited two sections in the eastern and western parts of the Naukat depression and succeeded in bearing out their structure. These sections are of a rather peculiar nature, and differ strongly from the normal sections of Ferghana, which induced the author to publish the new data obtained.

The first section is located near the Yalpak-tash village on the road between the town of Osh and village Iski-Naukat. A brief characteristic of this section is given below.

B u k h a r a s t a g e. The Palaeogene begins with a member of white gypsum with separate thin bands (each 10—30 cm thick) of green clays, white marls and porous dolomitized limestones. No fauna was found here, but in the next section in limestone bands typical fossils of the Bukhara stage were discovered. Thickness of the gypsum member, 40 to 50 m.

S u z a k s t a g e. Gypsiferous clayey member, pink and greenish in colour, with two beds of gypsum (0.5 and 1 m) in the upper half. Thickness, 8—10 m.

A l a i s t a g e. Light-grey and yellowish limestones, thick and indistinctly bedded. Among them occur dolomitized farinaceous horizons, shell limestones, with a profusion of *Turritella alaica* Vial. et Solun and indeterminable pelecypods, as well as oyster beds consisting of accumulations of *Ostrea turkestanensis* Rom. At the base of the limestones there is a layer of basalt conglomerate (0.5 m) of poorly rounded fragments o Palaeozoic rocks up to 5 cm in diameter, scattered rather sparsely in an areno-calcareous cement. Thickness, 16 m.

T u r k e s t a n s t a g e. Light-grey, greenish, highly marlaceous clays. 1.5 m. Dark, bordeaux-red clay with small quartz grains. 1.0 m.

Rishtan and Khanabad stages. Gypsum contaminated with red clays, with streaks of red clay sandstones and clays. 15 m.

Sumsar stage. An alternation of brownish-crimson sandy clays or highly argillaceous sandstones and grey-pink medium-grained loose sandstones. Layers of pure clays occur, too. The less the quantity of sandy material contained in the clays, the deeper is their colour. 50 m.

Massaghet stage. An alternation of brownish-red sandstones, mainly coarse-grained, with brownish-grey rubbly conglomerates and breccias. A subordinate place is occupied by highly argillaceous rather loose sandstones and sandy clays. Up to 250 m.

It is no easy matter safely to separate definite stages within the section. It is possible after all that not the whole member of lower gypsum belongs to the Bukhara stage, but that part of it is Danian. As well known, in North-Eastern Ferghana the Bukhara stage is represented by a similar gypsum member with thin intercalations of limestones containing a typical fauna. This member is underlain by brown-red clays of the Danian stage. In Western Ferghana the facial composition of the Bukhara stage represented by a normal marine facies is different; it consists of green clays and sandstones crowned with a bed of limestone or dolomite with *Corbula*. The Danian is composed of Goznau gypsum. In the sections of the Naukat depression both the independent member of Goznau gypsum and the radiolite horizon underlying the red Danian series in the north-east are absent. These sections in the boundary zone between the Cretaceous and the Palaeogene are as it were of intermediate nature between the southern (Isphara) and the north-eastern (Kizyl-Yar) type. The possibility is not excluded that the Goznau gypsum and that of the Bukhara stage in this part of Ferghana, which holds an intermediate position between the areas of development of the two types of the section, are fused into a single gypsum formation. It may be, however, that the entire gypsum formation refers to the Bukhara stage, and the Danian stage comprises the upper portion of the underlying red series.

The presence of the Sumsar stage in the section described has not been proved faunistically. The only reason for referring the crimson and the pink areno-argillaceous rocks to the Sumsar rather than to the Massaghet stage lies in the sufficiently distinct difference in colour. All over Ferghana the Sumsar stage is represented by crimson rocks, and the Massaghet stage, by brick-red ones. A similar colour difference along with the presence of fauna in another section described below justifies the assumption of the presence of the Sumsar stage here, too.

Especially obscure is the question as to the position in the section of the lower parts of the Upper Ferghana subdivision.

To this the author refers the intermediate reddish gypsum member lying between the Sumsar crimson rocks and the Turkestan stage. Such a kind of section of the Upper Ferghana subdivision is unusual for Ferghana. It may only be said that in the more easterly portions of the Alai range some part in this subdivision is played both by gypsum and by red-coloured rocks. In the Tadjik depression and in the eastern part of the Alai valley the Rishtan stage is represented by gypsum and by dark-red clays. Such remote analogies are, certainly, insufficient to form a definite idea as to the age of the formation discussed, but on the basis of the bulk of evidence available, the reference of this formation to the Upper Ferghana subdivision seems to be permissible.

The other section is located on the Abshir River, near the Burbash village.

Here the Upper Cretaceous member of red clays with layers of light-coloured greenish sands is overlain by a gypsum member, about 10 m thick. Higher up one may distinguish a member consisting of pale-pink and greenish-grey silty clays with thin streaks of gypsum. Its thickness does not exceed 3 m. The age of the gypsum is somewhat uncertain. In the author's opinion it seems most probable that they correspond to the Goznau gypsum of the Isphara type of sections and belong to the Danian stage. The pink and grey clays may be still Danian, or may possibly belong to the Bukhara stage.

The Bukhara stage, leaving no room for doubt, faunistically characterized, is represented by a member of white gypsum with streaks of white porous dolomitized limestones up to 1 m thick. In these limestones the author has found a typical fauna of the Bukhara stage: *Corbula* (*Cuneocorbula*) *angulata* Lam., *C. biangulata* Desh., *C. asiatica* Vial. and *Modiola jeremejewi* Rom. Thickness, about 40—50 m.

The Suzak-stage is very poorly exposed. Judging by the talus between the Bukhara gypsum and the limestone Alai ridge, it consists of a pink gypsiferous clayey member, whose thickness does not exceed 8—10 m.

Alai stage. A limestone member (about 20 m) forming a continuous high ridge stretching over a large distance.

Turkestan stage. Green clays, with a profusion of *Fatina esterhazyi* Pav. and *F. böhmi* Vial., 3 m thick. Higher up lies white, occasionally yellowish shell limestone, containing a profusion of natural casts of pelecypods; rarely indeterminable small oysters are encountered. Thickness, 0.70 m.

In another exposure, approximately 2—3 km further east, the lower part of the Turkestan stage consists of a continuous oyster bed with *Fatina*, 5 m thick. The thickness of the yellow shell limestone overlying it also increases to 1.5 m. It is in this exposure that higher lying parts of the section were observed.

Rishtan, Isphara and Khanabad stages. An insignificant unit of green and grey clays, only 4.5 m thick. It is very poorly exposed, and it has not been possible to establish the boundaries of the separate horizons without deep diggings.

In three diggings: a) 1 m, b) 3.5 m and c) 4 m from the roof of the Turkestan shell limestone the following rocks have been found: a) light-green plastic clays; b) dark-grey, nearly black clays; c) green plastic clays. In the dark-grey clays *Radiolaria* have been found, the presence of which in Ferghana is known to be characteristic of the Isphara stage. The overlying green clays (c) do not differ in outward appearance from the common clays of the Khanabad stage. The light-green clays underlying the grey ones refer probably to the Rishtan stage. In the talus some specimens of *Ostrea tianschanensis* Rom. were found, proving the presence of the Khanabad stage.

The Sumsar stage begins with an insignificant bed of variegated sandstone, overlain by plastic crimson clays typical of the Sumsar stage of the entire Ferghana but having here an insignificant thickness (only 4—5 m). The presence of *Gryphaea* (*Ferganea*) *sewerzowi* Rom. is evidence in favour of their Sumsar age.

The Massaghet stage consists mainly of fine-rubble conglomerates, gravel beds and sandstones, light-red and pink in colour, of a thickness of about 60 m.

Sokh series. The red rocks of the Massaghet stage are unconformably overlain by the brownish-grey rubble conglomerates and breccias probably belonging to the Sokh series (stage). The angle of unconformity distinctly seen in the exposure attains 15°, the Massaghet beds dipping south at an angle of 60°, and the grey conglomerates dipping also south at an angle of 45°.

The section of the Palaeogene on the Abshir River is very interesting, in the first place because taken as a whole it is unlike any principal type of the Ferghana sections. At the same time the Bukhara stage is built according to the Kizyl-Yar type (N.-E. Ferghana), and the Suzak stage rather after the Isphara type (South Ferghana), but is characterized by an insignificant thickness. The Turkestan stage terminates by an analogue of the *m* horizon (according to the nomenclature of K. P. Kalitzky), and the Upper Turkestan clays are absent, as it is the case in a number of sections of the Kurama range. Finally, the Upper Ferghana division in its general character is similar to that of the Tashkent region (Beldersai type of the left bank of the Chirchik River).

There is no doubt that the Abshir section occupies a marginal position. During the Massaghet time the area of destruction was located nearby, evidence of which is presented by the development of rubble conglomerates and breccias. If these

Massaghet rocks be compared to the thin Massaghet marly clays of the Isphara section, it will be at once clear that the latter were deposited much farther north from the mountain massif rising at that time, while the formation of the former occurred somewhere near its very foot.

It is of interest to note the considerable differences between the Upper Ferghana beds of the Abshir and the Yalpak-Tash section, and the similar structure of the deeper deposits of the Palaeogene. The Yalpak-Tash section refers to a much shallower zone of the Upper Ferghana sea and in its nature is related more closely to the extreme south-east of Ferghana and of the Alai range.

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MINERALOGY

BOEHMITE FROM THE NEPHELINE PEGMATITES OF THE VISHNEVYE MTS. IN THE CENTRAL URALS

By E. M. BOHNSTEDT-KUPLETSKAYA and N. I. VLODAVETZ

(Communicated by D. S. Beliankin, Member of the Academy, 18. VII. 1945)

While working in the Vishnevye Mts. during the summer of 1943, one of the authors discovered in large natrolite blocks in the lower quarry of a feldspar mine at Kurochkin Log small light-yellow crystals accumulated in cavities. At first sight these crystals were mistaken for zeolite. Studied in more detail, however, they have been found to be boehmite which has been for the first time observed in crystals open to direct examination and in a paragenesis uncommon to boehmite. Small crystals of boehmite are always gathered in druses, forming crusts several millimeters across. They have segregated on small needles of natrolite, partly cementing them. In the same cavities are observed beautiful water-transparent crystals of hydrargillite. In one of the natrolite blocks calcite crystals have been noted in the cavities along with boehmite and hydrargillite. In some cavities the boehmite is coated by light-yellow halloysite—product of a hypergene alteration of the minerals, which did, however, not affect the boehmite itself.

As revealed by a more detailed study of the material collected, the boehmite is confined not only to cavities in natrolite, but is scattered in the shape of small crystals throughout the natrolite, composing jointly with the latter the so-called spreustein, formed as a result of hydrothermal alteration of nepheline. But while in the pegmatites of Langesund fjord W. Brögger observed hydrargillite-diaspore spreustein, in the pegmatites of the Vishnevye Mts. the entire spreustein is hydrargillite-boehmite streustein. It has been reported not only from large pegmatite lenses of Kurochkin Log, but also from other pegmatites of miaskites: on Mt. Karavai and in the Cheprunov Log.

The results of a preliminary study of the boehmite from the Vishnevye Mts. are given below. Boehmite forms small crystals (up to 0.2 — 0.3 mm) of a very characteristic lenticular shape, with an eminent cleavage along one direction. It is extremely difficult to separate out individual crystals for study, since they usually form more or less parallel intergrowths reaching several millimeters in size. Externally these crystals resemble very closely the boehmite crystals from the French bauxites described by Lapparent [1]. Only one face is brilliant, along which follows perfect cleavage—according to Lapparent, *p* (001), accepted by us as *b* (010); characteristic is also the presence of narrow faces of rhombic prisms (011) somewhat glittering. All other faces are not distinctly developed, rounded, and their surfaces are dull. A careful measurement of a number of crystals on the Goldschmidt two-circle goniometer made it possible, however, to obtain the angular values of the forms from reflexes and hardly distinguishable signals. The data obtained for prism (011) are quite significant: angle h (011) : b (010) = 58°25′; less reliable is the angle of the prism which could be measured directly only once: m (100) : b(010) = 67°30′; the remaining forms have been determined on the basis of imperfect measurements only, and calculations made with the aid of stereographic projection. Using the

above cited, although not very reliable angles obtained for $h(011)$ and $m(110)$, we calculated the axial ratio for the boehmite from the Vishnevye Mts. $a : b : c = 0.41 : 1 : 0.64$, very closely approaching the axial ratio of lepidocrocite $a : b : c = 0.43 : 1 : 0.64$, as given by Posnjak and Merwin (2).

As shown above, boehmite possesses an eminent cleavage along (010); on the cleavage plates under the microscope two directions of cleavage are observed, corresponding to (001) and (100); the latter is imperfect. The mineral is brittle and, unlike hydrargillite, can readily be ground in a mortar. Hardness, 3.5—4. Specific gravity, $d_{15}^{18} = 3.019$ (determined in the Thoulet solution). Lustre on the cleavage face, pearly; on the fracture, vitreous. Colour, light-yellow, light greenish-yellow to nearly colourless. Powder, white. In thin-sections, colourless; distinguished by perfect cleavage in one direction and low birefringence. Optic plane coinciding with the perfect cleavage plane (010). Crystals somewhat elongated along the c axis; $N_g = c$, $N_p = a$, $N_m = b$. Mineral biaxial, positive, with a large optic angle (about 80°). In immersion on cleavage plates $N_g = 1.661 \pm 0.003$, and $N_p = 1.646 \pm 0.003$ have been determined, whence $N_g - N_p = 0.015$.

Before the blow-pipe the mineral does not melt, becomes white and dull, and splits following the cleavage. The optic orientation of the mineral is changed in this case, $N_g = a$, $N_m = c$, $N_p = b$, and the index of refraction becomes lower, N_g about 1.609, N_m about 1.602. In the tube it gives off water; with borax the bend is sligthly greenish when hot, and colourless, when cold. In acids insoluble. With phosphorus salt and sodium carbonate fuses with difficulty, leaving a characteristic skeleton, the nature of which stands in need for further investigation.

For chemical analysis about 2 g of the mineral was carefully sorted out under the binocular microscope. The procedure was the one commonly used for silicates. The finely ground sample was fused with sodium carbonate. Fusion had to be repeated several times, the melt obtained being incompletely decomposed by hydrochloric acid. Water was determined directly in the Penfield tube.

Certain interest belongs to the method of gallium determination, for which reason it will be dealt upon here at some length. The sesquioxides were fused with potassium pyrosulphate and dissolved in 1.5 N HCl. Iron and gallium were precipitated with cupferron, but since in the presence of a large amount of aluminium the latter was also partly precipitated, precipitation with cupferron was repeated. The cupferron residue after the second precipitation was ignited in a quartz crucible, fused again with pyrosulphate and dissolved in water slightly acidulated with hydrochloric acid. The solution heated to boiling was neutralized with ammonia in the presence of methyl-red as indicator and then boiled for several minutes. The precipitate of gallium and iron hydroxides was filtered off, washed and dissolved on a filter with hydrochloric acid. Upon evaporating in a porcelain dish over a water bath to a thick syrup-like consistency the hydrochloric acid solution was diluted with a small amount of 6 N HCl. The separation of gallium and iron from the aluminium possibly still present was achieved by extracting the two first elements with ethyl ether saturated with HCl. Gallium was separated from iron by the tannine method (3).

Negative results were obtained with some of the determinations made by the routine methods of chemical analysis of the following elements: beryllium, rare earths, titanium, niobium and tantalum, zirconium, sulphur, and also of phosphoric acid.

The analysis showed the following composition of boehmite:

	Per cent	Molecular ratio	
Al_2O_3	81.60	0.8005	
Ga_2O_3	0.05	0.0003	
Fe_2O_3	0.58	0.0036	1.00
CaO	0.26	0.0046	
SiO_2	2.11	0.0351	
H_2O	15.31	0.8498	1.01
Total . . .	99.91		

The results of analysis show:

1. An unusually high gallium content, which is in agreement with its high concentration in the nepheline from the Vishnevye Mts., in two specimens of which Tolmachov and Filippov([4]) spectroscopically determined 0.01 per cent gallium in each. An increased gallium content is known for another modification of the aluminium monohydrate, the diaspore. According to Goldshmidt and Peters ([5]), the content of Ga_2O_3 in the diaspore from Chester and Langesund fjord is 10^{-2} and 10^{-1} per cent, respectively.

In the bauxite from Gravia (Greece), consisting mainly of diaspore, $3.7 \cdot 10^{-2}$ per cent Ga_2O_3 has been found. It seems hardly judicious to explain the increased gallium content in diaspore by the similarity of its crystalline structure with that of gallium hydroxide, as it had been suggested by Böhm and Kahan ([6]), since gallium content of the same order is observed in the case considered here in boehmite with a lepidocrocite structure.

2. The molecular ratio of $Al_2O_3 : H_2O = 1 : 1$, corresponding to the simplest formula of aluminium monohydrate, cannot be obtained unless all the oxides except water are referred to the same group with Al_2O_3.

3. The mode of occurrence of silica remains obscure; it may either be present as a mechanical admixture or enter into the composition of the boehmite itself.

Further studies of a larger material, which are in progress, will enable us to give a final characteristic of the boehmite from the Vishnevye Mts. and of the minerals closely associated with it.

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REFERENCES

[1] J. Lapparent, Bull. Soc. Fr. Minér., **53**, 261 (1930). [2] E. Posnjak and H. Merwin, Amer. J. Sci., **47**, 311 (1919). [3] Н. И. Владавец, Тр. Ин-та геол. наук, **17**, 12 (1940). [4] J. M. Tolmachov and A. N. Filippov, C. R. Acad. Sci. URSS, III, 366 (1934). [5] V. M. Goldschmidt u. Cl. Peters, Nachr. Ges. Wiss. Göttingen, Math.-phys. Kl., 177 (1931). [6] J. Böhm u. G. Kahan, Z. anorg. Chem., **238**, 350 (1938).

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PETROGRAPHY

ON THE CRYSTALLINE PHASE IN COALS

By A. S. KUMPAN

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When classing coals with definite genetic groups and determining the degree of their metamorphism, various methods have been used among which the most important are chemical analyses, microscopy, and determination of the physical properties of coals. Questions bearing upon the internal structure of coals have not been worked out in detail, and structural analysis of coals has correspondingly found but insignificant application.

The use of the powdergram method in the study of natural coals by Mahadevan([1]), Boldyrev and Kovalev ([3]), Turner and Anderson ([2]) made it possible to reveal in coals of a high degree of carbonization, mainly in anthracites, the presence of graphite crystallites. The study of stone coals and brown coals gave but conflicting results. While some investigators (Boldyrev and Kovalev) deny the crystalline structure in stone coals, others, as, for intance, Sedletzky and Brunovsky ([4]), attribute a crystalline structure even to humic acids.

It should be noted that such a controversy in opinion is largely due to the fact that specimens of coals for study were taken without reference to the geological conditions and regardless of the degree of their metamorphism. As a result, the attempts of Mahadevan to make out the difference in the internal structure of various petrographic ingredients (vitrain, fusain, durain) proved a failure, since these ingredients had been separated out from coals without taking into account the degree of their carbonization.

From this it may be seen that the internal structure of coals should be considered in the process of its formation and alteration caused by metamorphism, related with their transition from one genetic type to another.

The work by Bone has shown that the cyclic structure of humic acids is retained in humins irrespective of the degree of carbonization of coals, and is not destroyed under the action of high temperatures. While studying chlorinate coals, Eccles and McCulloch arrived at the conclusion that various humins, though retaining a similar structure of their nuclei, differ in the side chains only. Experiments on the artificial production of coals (Berl, Gropp and Bode) have shown that, irrespective of the composition of the initial material, in the process of carbonization these coals obtained an aromatic structure. Orlov ([5]) arrived at the conclusion as to the possibility of a conversion in the process of carbonization of compounds of the furain series, characteristic of cellulose, into aromatic compounds. Berl considered that in the process of carbonization condensed systems are formed, and direct bonds between carbon atoms may arise, which leads to the formation of a graphitic structure.

Thus, the change of the original vegetable matter in the process of humification and metamorphism (carbonization) may result, irrespective of the composition of the starting vegetable matter (cellulose or lignin),in the formation of a free

carbon molecule, in which, upon splitting off the side chains, the carbon atoms become joined directly to each other.

The formation of free carbon atoms leads to the formation of the crystalline lattice in coals.

The subsequent evolution of free carbon brings about the appearance of special inorganic (polar) bonds connecting the planes of the hexagon with one another; the stabilization of these bonds leads to the formation of the crystalline lattice of graphite. The appearance and development of the polar bonds produce such properties of coals as anisotropy, electrical conductivity and heat conduction. Naturally enough, the above-described process is only a scheme complicated by the conditions of genesis and metamorphism of the coals. This scheme, however, may be assumed to underly the structural conversions of coals which ultimately lead to its chemical and physical properties.

As the object of study was used the collection of coals from the Donetz Basin assembled by E. O. Pogrebitzky in the course of his investigation of the metamorphism of coals. Studied was, moreover, coal from two seams of the Karaganda Basin—the upper («New») and the lower («Double») one, as well as of specimens of coal from the Noghinsky and Bredinsky deposits, altered under the influence of intrusions of igneous rocks. For the sake of comparison shungites and Ceylon graphite were also investigated.

The X-ray analysis was carried out at the laboratory of the All-Union Geological Institute by G. A. Kovalev. The subsequent photometry of the powdergrams was made by V. N. Protopopov. The chemical analyses were carried out at the laboratory of the above-named Institute. The results are summed up in Table 1.

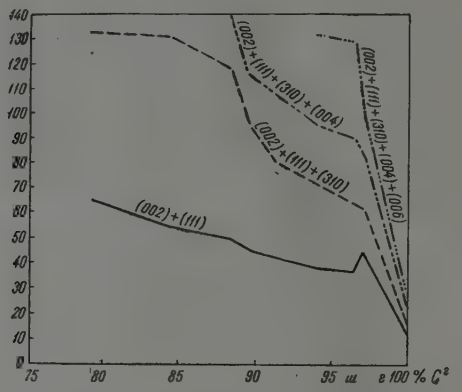
Table 1

Values of Total Width of Maxima in mm

Place of sampling	No. of specimen	Organic C content, %	Symbols of faces			
			002+111	002+111+310	002+111+310+004	002+111+004+006
Aktiubinsk region, Kurasha-Sai deposit, test-pit 22, seam II	71	62.61	48	—	—	—
Donetz Basin, Lisichansk region, Rukhimovich shaft, horizon 198, seam L_3	218	76.96	64	132	—	—
Sorokin region, shaft 2 «Proletarka», 4 drift, seam m_3	59	84.43	55	130	—	—
Sorokin region, shaft 2, 29 western drift, seam K_5	51	88.53	50.5	118	139.5	—
Sorokin region, shaft 3, 8 drift, seam K_5	85	89.71	47.5	94	115.0	—
Voroshilovsk region, shaft 9, seam K_5	166	91.23	43.0	81	108.5	—
Makeyevka, Chicherin shaft, incline 3, seam K_5	38	94.12	39	71	96.0	131.0
Dolzhansk region, Schwarz shaft, 26 drift, seam K_5	96	96.36	38	64.5	90.0	129.5
Leningrad region, Shunga village, Shungite of the 1st variety	Shungite	97	44	62.0	81.6	96.0
Ceylon, Stragau deposit	Graphite	100	10	17.5	22.5	25.5

In order to compare the results obtained and to determine the nature of the structural change from coals with a low degree of carbonization to anthracites and graphites, measurements were made of the width of the maxima on the photo-

grams, the width of the maxima being successively summed up with the appearance of new faces. The results are shown in Table 2 and on the figure, in which along the ordinates was plotted the total width of the maxima, and on the abscissae,



Variation in total width of maxima as dependent upon the increase in hydrogen content. *ш*, shungite; *г*, graphite

carbon content per combustible mass. For sake of comparison, on the diagram and in the tables are also given the results of a study of the brown coals from the Aktiubinsk region.

Table 2

Values of the Width of Maxima for Karaganda Coals

Seam	Width of maxima, mm			Total width of maxima, mm	
	002	111	004	002+111	002+111+004
New (Novy)	24.5	30.0	21.0	54.5	75.5
Double (Dvoinoy)	22.0	19.0	23.0	41.0	64.0

From an analysis of the results obtained the following conclusions are to be made.

1) With an increase in the carbon content from long-flame coals to anthracites and graphites, the crystalline lattice is stabilized and perfectioned, which is manifest in the appearance of new faces and in a decrease in the total width of the maxima.

2) The sharpest changes in the structure take place at the boundary between anthracite and graphite. A similar critical point may be established between stone-coal and anthracite.

3) The crystalline phase in brown coals is not distinctly pronounced, this type of coal requiring additional investigation.

A comparison of the photograms with the powdergrams of coals from the New (Novy) and Double (Dvoinoy) seams of the Karaganda section (Table 2) shows a decrease in the total width of the maxima for the lower, Double seam, as compared to the New seam, the number of faces revealed being equal.

A comparison of the photograms of the coals from the Bredinsky deposit with specimens of coal from seam 14 (Nos. 2 and 3) at the contact with thin quartz veins, and No. 4, from the middle of the same seam, not crossed by thin quartz

veins, shows a sharp increase in the width of the maxima for the metamorphosed portion of the seam (Table 3).

Table 3

Values of the Width of Maxima for the Noghinsk and Bredinsk Deposits

Place of sampling	No. of specimen	Width of maxima, mm				Total width of maxima, mm		
		002	111	310	004	002+111	002+111+310	002+111+310+004
Noghinsk deposit, 30 m from the contact with diabase . .	700	28	17	20	46	45	65	111
Bredinsk deposit, seam 14, at the contact with thin quartz veins	2	18	17	17	27	35	52	79
Do	3	17	15	17	30	32	49	79
In the middle of the seam, without thin quartz veins .	4	20	24	72	27	44	116	143

The preliminary results of investigation show that structural X-ray analysis, in conjunction with chemical and microscopic investigations may serve as a method for determining the degree of metamorphism of the coal and for classing it with a definite genetic group.

A further development of this method may furnish guiding evidence for the classification of coals and determination of their economic value.

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REFERENCES

[1] C. Mahadevan, Fuel in Science and Practice, XIV, No. 8 (1935). [2] H. Turner and H. Anderson, ibid., XI, No. 7 (1932). [3] А. К. Болдырев и Г. А. Ковалев, Зап. Ленингр. горного ин-та, 10, в. 2 (1937). [4] И. Д. Седлецкий и Б. К. Бруновский, Тр. Ломоносовск. ин-та АН, VIII, № 51—59 (1936). [5] Н. А. Орлов, Очерки по химии угля, изд. АН СССР, 1934.

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HISTOLOGY

VARIATION OF SKIN ELEMENTS IN KARAKUL LAMBS

By G. G. TINIAKOV and A. P. TINIAKOVA

(Communicated by L. A. Orbeli, Member of the Academy, 1. VIII. 1945)

The object of the present investigation was to find out with the aid of histological methods the cause of asymmetry and other abnormalities in the development of skin in Karakul lambs. Fragments of skin were taken for examination from 19 individuals of robust constitution in March and early in April, 1942. The lambs were obtained from the Kara-Kum pedigree collective farm, in Uzbekistan.

Table 1

No. of lamb	Age in days	Side	Character of wool covering	Thickness of epidermis in μ	Depth of coarse hair roots in μ	Depth of fine hair roots in μ
1	1	Left	Flattened	22	1 801	812
		Right	Valek	20	1 735	856
2	1	Right	Valek-bean	21	1 587	1 013
3	2	Left	Valek	22	1 506	749
		Right	Bean, slightly curled	22	1 675	574
7	1	Left	Valek	17	1 408	648
		Right	Bean, loose	19	1 626	684
19	1	Left	Valek	—	—	—
		Right	Roan	18	1 722	777

As may be seen from Table 1, all the individuals showed either on the left or on the right side of their bodies a perfect type of the curl «valek» A histological analysis of skin fragments taken from different animals from the side where their fleece showed this pattern, revealed a picture which is usually observed in this case (Fig. 1). In cross-section the hair roots were arranged in a regular row, all of them growing in the same direction. The hair bulbs were usually situated at the same level and proved of about the same size. The bases of the sweat-glands which were found to belong to the tubular type were situated at the level of the hair bulbs or even somewhat deeper.

Unlike this, skin areas corresponding to the vicious curl pattern showed sharply pronounced irregularities in the arrangement of the hair roots; the latter were situated at different levels and were but poorly developed (Fig. 2).

Among the vicious characters of the fleece Karakul-sheep breeders class also the appearance of roan coloration. This character was displayed in our material by lamb No. 19. Its fleece was an intense black and of a perfect «valek» pattern

everywhere, except the area where it was roan, while the latter was covered with three kinds of hairs, about a half of which were black, a much smaller number grey and quite a negligible number, white. The longest proved the grey hairs and part of the white ones: they made up the tip of each curl in the form of silvery, Sur-like braids.

A peculiar feature inherent in the grey hairs was their white core, regularly scattered with oblong or rounded grains of black pigment. Within such areas where the pigment grains are but few in number, the hair appears mottled-white, whereas it looks grey where they are abundant. The pigment grains are scattered not only throughout the thickness of the hair, but also in its epithelial sheath. The grey hairs are the thickest; the white ones, the thinnest. The roots of the white hairs penetrate to a deepest level, attaining an average depth of 1722 μ. The

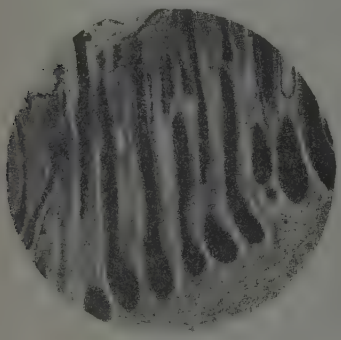

Fig. 1

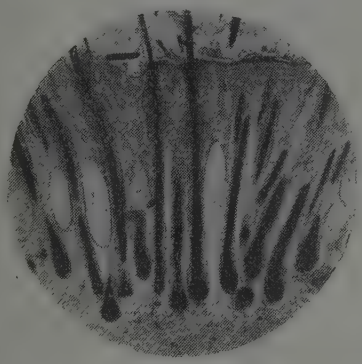

Fig. 2

level of the base of the sweat-glands does never attain that of the bulbs of coarse hair roots.

An analysis of the fleece of a number of cells of the «Sur» type has shown that the roan spot is a character dominant in this breed. By the asymmetry (1) Karakul-sheep breeders usually understand a flattening of the curl which is mostly observed on the left side of the animal and is accounted for by purely mechanical factors. It does not seem improbable that among our material flattening and depression were also due to mechanical influences. When viewed from the biological standpoint, however, any character which appears on the one side of the body can be qualified as an asymmetry. It has already been pointed out that all signs of asymmetry in the pattern of the wool covering corresponded to such areas of the skin as differed sharply in their structure from the rest of the skin. But these differences were in full agreement with the genotype of the skin whose wool covering corresponds to that of the asymmetrical area. We believe therefore that many of the asymmetrical variations of the features of the wool covering in lambs, which are connected with profound structural alterations of the skin, must be due to locally operative genotypical, rather than to purely mechanical agents. Thus, a purely mechanical agent is scarcely able to produce the phenomenone of roan coloration which is accompanied by a complex structure of the hair root system.

Overgrown Hairs. There are cases where not isolated body areas but the animal as a whole is affected by a vicious character. This is the case of lambs that are born with overgrown hairs, a character which utterly deteriorates the pattern of their fleece. An analysis of individuals displaying this defect is given in Table 2.

From Table 2 it is easy to see that the character of overgrown hair was invariably associated with a mis-shapen bean-curl and other deteriorated curl-patterns. All such individuals are characterized by a vigorous development of the hair root

Table 2

No. of lamb	Age in hours	Character of wool covering	Thickness of epidermis in μ	Depth of coarse hair roots in μ	Depth of fine hair roots in μ
1	12	Bean, strongly overgrown . . .	21	2 397	624
5	12	Uncurled bean, ring. Hair overgrown	26	2 224	651
6	—	Ring and poorly curled bean. Hair overgrown	21	2 344	698

system. The roots of kemp hairs lie at a depth of more than 2 000 μ, which is never observed in normal individuals of this age. The shape of the bulbs of these hairs varies greatly. Noteworthy is also the presence of a large number of small hair roots, which is never the case in individuals which are not subject to the appearance of overgrown hair.

This character may possibly be due to conditions favouring a vigorous development of epithelial strands of the hair formed, and to an earlier formation of both large and small roots, as compared with normal individuals.

Comparative Stability of the Curl Pattern. A perfect pattern of the curl in lambs is, as a rule, preserved during a very short period of time only. This should in the first line be accounted for by the longitudinal growth of the hair and by the fleece growing grey. The process of deterioration of the original structure of the curl takes a course differing as to the individual animal. The different degree of the stability of the curl pattern should also be accounted for by the variation in the structure of the skin in these animals. Table 3 contains comparative data derived from an analysis of 8 lambs.

Table 3

No. of lamb	Age in days	Character of wool covering	Thickness of epidermis in μ	Depth of coarse hair roots in μ	Depth of fine hair roots in μ
11	19	Excellent preservation of pigment and lustre	25	1951	901
15	31	Valek and bean well preserved .	30	1794	864
12	20	Pea-bean. Pigment and lustre reduced	24	1621	776
13	28	Uncurled bean and corkscrew; pigment and silkiness reduced	38	2314	636
14	27	Bean partly tending to corkscrew. Pigment and silkiness preserved	30	2128	932
16	31	Bean uncurled, ring. Grizzled hair especially abundant on the rump	25	2330	1021
17	30	Fleece turns grey, lustre and pigment reduced	25	1854	862
18	30	Grey hairs appear. Pigment and lustre preserved	34	3011	988

When first examined for the type of their fleece, the lambs referred to in Table 3 had a high grade curl pattern. Histological analysis has shown that the two first lambs which preserved the original pattern of the curl had no fine hair roots. The next three specimens which failed to preserve the original curl pattern, are distinguished by a great variety of their hair roots. Many of them are strongly

bent in different directions, and Nos. 13 and 14 even show corkscrew-like roots. The loss of the original curl pattern which takes place towards the end of the first month may possibly be due to an intense growth of the finer, younger hairs which soon interfere with the initial structure of the curls.

The last three specimens are distinguished by the appearance of grey hairs. At this age grey hairs may appear among fine and fluffy wool only. The number of white hairs varies from 5 to 30 per cent, as to the individual. According to our observations, all the grey hairs do not arise by way of a gradual transformation of finer black hairs into grey ones, but are white from the very beginning.

Colour of the Fell. Grey colour in Karakul lambs is due to the presence in their fell of both black and white hairs. Our investigation has shown that the black, always coarser hairs of grey individuals are never mosaic; the light hairs are likewise white throughout their length, without any sign of mosaic coloration. The roots of both white and black hairs of Karakul sheep have been found to be situated at a similar level. Nearer to the skin surface are situated the smallest roots of black and white hairs. Black and white kemp hairs seem to form at the same time in grey Karakul sheep.

In all coloured individuals the hair roots are situated much deeper than in black specimens of the same age. As well known, pink coloration is due to a mixture of red, brown and white hairs. In specimens of this type of coloration, the most powerful are the roots of red hairs; against this background are scattered separate grains of black pigment. Less powerfully developed are the roots of white hairs which contain no grains of black pigment.

The root system in Kombar-sheep consists of two types of hairs. In light-chestnut Kombars powerful roots with well differentiated bulbs contain intense light-brown pigment. In dark-brown Kombars the roots are of the same type, but dark-red, grains of black pigment being scattered throughout the length of the hair. It is to these hairs that the lambs of this type owe their dark-brown colour. To the second type of roots belong the finer light hair roots situated at various higher levels.

An analysis of the colour characters went to confirm the conclusion based upon our observations on the appearance of grey hairs, *viz.* that the character of the coloration of fell in lambs depends upon the process of hair formation in the bulb. None of the colours results from a gradual transformation of the one hue into another. Consequently, the fundamental cause responsible for the character of the coloration of the hair lies in the specific genotype of those intensely developing epithelial cells from which is formed the primordial tubercle of the embryonic hair.

Under these conditions the colour type of an individual hair may well depend upon the specific genotype of a single cell. And the difference between the genotypes of individual cells may be due to nothing else than to processes of somatic mutation.

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REFERENCES

[1] М. Ф. Иванов, Овцеводство, 1940.

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GENETICS

RELATION BETWEEN AGE AND MUTATION PROCESS IN *DROSOPHILA MELANOGASTER*

By J. M. OLENOV

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Studies demonstrating the possibility to produce mutations by acting directly upon sex cells were of high positive value. They failed to solve, however, and could not solve, indeed, the problem of the causality of the so-called spontaneous mutations; up to the present day there exists no unanimous opinion with regard to this problem. According to Timofeeff-Ressovsky([1]), spontaneous mutations represent intramolecular changes whose probability is dependent upon purely physical causes having no relation to the life of the cell. The number of mutations produced is a function of time and does not depend upon the number of generations studied. Certain data which are incompatible with this view will be discussed below. Here we shall only remind Navashin's brilliant investigations ([2], [3]), which have shown on ageing seeds the decisive effect of the physiological condition of the organism, as determined by the surrounding medium, upon the mutation process. Later experiments went to demonstrate that this conclusion holds true for gene mutations as well ([4],[5]).

We studied the relation between the mutation process and age in males of *Drosophila melanogaster*. Like in the experiments of Timofeeff-Ressovsky, the males were kept without females in our experiments. The temperature during the experiments was 22° C. One-day-old males served as control. Owing to the aseptic culture technique after Guenot ([6]) adopted by us (ordinary yeast medium served as food), the males survived to the age of 4—4.5 months, which is three times that attained under ordinary conditions. However, the 4—4.5-month-old males were completely sterile; their sterility was caused by the death of the sperms or the loss of their activity ([7]) rather than by the general decrease in the physiological activity of the individuals, coitus having been repeatedly observed. Therefore, for the analyses of the mutation process only 2.5—3-month-old males were used.

In the preliminary experiment we studied, under the application of the C1B method, 408 chromosomes belonging to 2.5-month-old males from the forked line characterized by a low mutation rate. There were found 4 lethal mutations ($0.98 \pm \pm 0.48$). However, our subsequent work did not confirm these results.

We carried out an extensive study of the spontaneous mutation process in 2.5-month-old and control males belonging to the same forked line. Lethal sex linked mutations were registered with the aid of the C1B method (see table, experiment 1). Semilethals were discarded, like in all the previous experiments. The result of this experiment, *viz.* the absence of any difference between the mutation process in 2.5-month-old males and in the controls, stands in need for verification and explanation. Although nobody has ever demonstrated the existence of the germinal selection as a factor affecting the number of the lethal mutations found, its possible rôle ought to be brought out in the first place. We took again 2.5-

Experiment	Age	Line	Chromosome	Number of males	Number of chromosomes analysed	Number of lethals	Per cent of lethals	
1. Spontaneous mutations .	Control	Forked	X	107	2131	1	0.05±0.04	
	2.5 months	»	»	86	4225	2	0.05±0.03	
2. Roentgen-mutations . . .	Control	»	»	55	446	13	2.91±0.79	
	2.5 months	»	»	45	434	14	3.20±0.84	
3. Spontaneous mutations .	Control	White	II	98	1090	10	0.91±0.28	Diff. = =0.44± 0.35
	1 month	»	»	326	2952	40	1.35±0.21	
	3 months	»	»	284	2906	21	0.72±0.16	
4. Spontaneous mutations	Control	Uman 1938	X	41	1027	11	1.06±0.31	Diff. = =0.54± 0.36
	2.5 months	» »	»	62	1528	8	0.52±0.18	

month-old males and one-day-old males from the forked line and treated them with X-rays (the dose being 1000 r); after 8—10 days the irradiated flies were crossed with females C1B. If germinal selection were more strongly operative in aged males, the results of the experiments would be affected by this difference. As a matter of fact, the actual results did not confirm this hypothesis (see table, experiment 2).

The next experiment involved males from the line white. They are distinguished by their higher mutability as compared with the forked line referred to above. The spontaneous mutation process was studied with the aid of the Cysp method in one-day-old, one-month-old, and in three-month-old males. The aim of this experiment was: 1) to check up the results of the first experiment, 2) to study the possible rôle of germinal selection with the aid of an additional method, and 3) to trace the possible effect of age upon the mutation process.

This experiment (see table, experiment 3) revealed a slight, statistically insignificant increase in the mutation rate in one-month-old males as compared with the controls. No traces of this increase were found to persist in 3-month-old males. In the 4th experiment we studied again, by the C1B technique, the mutation process in the X-chromosome in a different line, the highly mutable line from the Uman population. The results obtained (see table, experiment 4) showed a slight statistically insignificant decrease in the mutation rate in 2.5-month-old males as compared with the controls.

The results of this study may briefly be summarized in the following way: 1) There is no general regular increase in the spontaneous mutation rate in 2.5-month-old or 3-month-old males as compared with the one-day-old, either in the X-chromosome or in chromosome II. 2) A slight statistically insignificant difference was observed in one-month-old males. It seems possible that in some cases a higher variation in the mutation rate occurred in aged males (preliminary experiment). 3) In 2.5-month-old males there was observed a statistically insignificant tendency towards a decrease in the mutation rate. 4) The results obtained cannot be interpreted in the light of the germinal selection hypothesis.

In order to give a correct interpretation to the data obtained, a careful comparative consideration is necessary. We showed in our paper on the mutation process in *Drosophila melanogaster*, occurring under the conditions of B_2 avitaminosis([11]), that the number of mutations arising in the sex cells of an individual up to the close of metamorphosis did not depend upon the duration of the development, its value being characteristic of that period of the individual's life. The most probable

explanation of this fact is offered by a suggestion made by Koltzoff [12], *viz.* that the spontaneous mutations are «errors» in the process of the reproduction of genes, *i. e.* their number should be related to the number of the cell divisions in the process of spermatogenesis.

Now if we proceed to an analysis of the data presented above, the following picture may be traced. During the process of spermatogenesis every cell undergoes a definite, although perhaps not quite fixed number of divisions. Inasmuch as spontaneous mutations arise in the process of the reproduction of genes, the number of mutations occurring in old males should not differ perceptibly from that in the young ones [13].

The process of the reproduction of the genes, however, is evidently not the sole source of spontaneous mutations. Sacharov and Shapiro [14] have shown that mutations arise in the mature sperms of *Drosophila melanogaster* while in the spermatotheca of the female, —a phenomenon which is probably analogous to the mutation process in the ageing seeds of plants. The increase in the number of mutations in 20-day-old males kept without females, as stated by Timofeeff-Ressovsky, and the corresponding tendency recorded for our one-month-old males of the line white, are evidently due to the accumulation of such mutations. Later on, however, a mass death or inactivation of the mature sperms sets in, leading ultimately to a complete sterility, so that in 2.5—3-month-old males the number of mutations becomes indistinguishable from that in the control. As to the decrease in the mutation rate in 2.5-month-old males of the highly mutable Uman line, as well as in 25—30-day-old males of other lines [13], one may believe that a more perfect knowledge of the peculiarities of these lines will enable us to give an explanation to this phenomenon.

At the present time it must be stated, however, that external influences have already been found either to increase or to decrease the number of mutations [15] arising in highly mutable lines.

The following principal conclusion may be drawn from the above said: it is both inadmissible on theoretical grounds and incompatible with factual data already secured to consider the process of spontaneous mutation as being independent on the life of the cell and the organism as a whole.

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Received
27. V. 1945.

REFERENCES

[1] N. Timofeeff-Ressovsky, Wiss. Forsch. Reihe, **42** (1937). [2] M. Navashin, Arch. wiss. Bot., **20**, 4 (1933). [3] М. С. Навашин и Е. Н. Герасимова, Биол. журн., **4**, 4 (1935). [4] J. D. Cartledge and A. F. Blakeslee, Proc. Nat. Ac. Sci., **20**, 2 (1934). [5] H. Stubbe, Wiss. Woche zu Frankf. a M., **1** (1934). [6] F. L. Guenot, Bull. Biol. Fr. et Belg., **51**, 1 (1917). [7] М. Е. Нейгауз, Биол. журн., **7** (1938). [8] J. Schulz, Radiation and the study of mutation in animals, **2**, 1936. [9] M. E. Lobashov, C. R. Acad. Sci. URSS, II, No. 5 (1934). [10] М. К. Скарбан, Зб. праць з генет., **2** (1938). [11] J. M. Olenov, Am. Nat. (1941). [12] N. K. Koltzoff, Les molécules héréditaires, 1939. [13] A. I. Zuitin and M. T. Pavlovetz, C. R. Acad. Sci. URSS, XXI, No. 1—2 (1938). [14] В. В. Сахаров, Ж. общ. биол., 1, 2 (1940). [15] Zamenhof, Nature, **153** (1944).

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GENETICS

HETEROCYCLICITY OF THE SYSTEM «MATERNAL CHROMOSOME — DAUGHTER CHROMOSOME»

By A. A. PROKOFIEVA-BELGOVSKAYA

(Communicated by I. I. Schmalhausen, Member of the Academy, 20. X. 1945)

The study of the behaviour of the nuclear elements in young and aged tubers of potato (varieties «Early Rose» and «Mayka») made it possible to reveal the changes that occur in the nucleus as the tuber grows older, and the differences existing between the daughter nuclei in the binucleate cells of the cortical parenchyme.

Along with the ageing of the tuber the resting nuclei of the cortical parenchyme undergo the following changes: towards the close of starch formation the nuclei lose their spherical shape, the nuclear contents (karyoplasm) suffer a sharp reduction in volume, the turgor of the nucleus drops, the nuclear membrane shrinks and becomes wrinkled, forming folds on the surface of the nucleus. The remaining nuclear contents takes intensely a diffuse stain, while the nucleolus shows but a slight staining. The basophily of the chromonemas and of the chromomeres is greatly reduced. The nuclei of the cortical parenchyme of the immature tubers have regular spherical shape. The chromonemas and chromomeres form a delicately stained nuclear structure. The karyoplasm remains unstained. Mitosis takes place in the large cells of the cortical parenchyme owing to which binucleate cells are formed: the phragmoplast which develops within a small protoplasmic region surrounding the nucleus cannot dissect the large body of the cell filled with vacuoles, leucoplasts and starch. Consequently, the nuclei in the binucleate cells are «sister nuclei».

The sister nuclei contained in binucleate cells are, as a rule, equal in size. In many instances they resemble each other in structure. In a number of cells, however, the nuclei forming the binucleate systems clearly differ from each other in their state. These differences may be distinctly observed both in «resting» nuclei and in those which have entered mitosis. The differences between the nuclei forming binucleate systems involve the following structural features: 1) the nuclei may differ from each other in the number and size of nucleoli; 2) the nuclei may differ greatly in the degree of the basophily of their karyoplasm (staining with haematoxylin, gentian and after Feulgen).

When comparing these data with those relating to the behaviour of the nucleolus in the nuclear cycle in *Vicia*, *Crepis*, *Aloe* and *Allium*[1], the following idea suggests itself: the cyclic states of the two sister nuclei of the binucleate system differ from each other. One of the nuclei is older than its partner. The heterocyclicity of the nuclei of the binucleate system is revealed in a clearer form by the study of the rare instances of their mitotic division.

1. At the prophase one of the two nuclei contains thin intensely staining chromosome threads, while a knot of loose slightly basophilic threads is observed in the other.

2. At the metaphase the state of the chromosomes in the two sets differs greatly; while one of the complexes is composed of poorly staining, loose chromosomes of indistinct structure and diffuse outlines (Fig. 1, left), the chromosomes making up

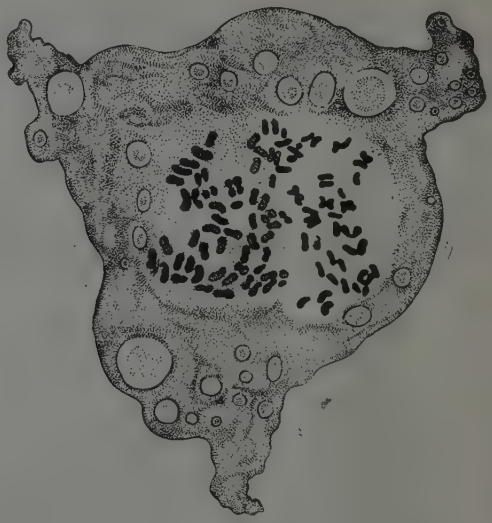

Fig. 1. *Solanum tuberosum*. Cortical parenchyme of tuber. Differences in state of chromosomes in metaphases of sister nuclei. × 1800

the other complex (Fig. 1, right) are strongly stained and thin, their structure being quite distinct. The whole portion of the cell containing the set of «loose» chromosomes is diffusely stained; on the other hand, the cytoplasm bearing the other, structurally differentiated complement remains entirely clear.

3. At the anaphase the chromosome cycles in the sister nuclei proceed in a different way. When the chromosomes belonging to the one complement have already reached the poles, the chromosomes forming the other complement are still at the early anaphase.

Two chromosome complements different as to their properties are thus formed in the mitosis, which results in the formation of a binucleate system: one of the two chromosome complements is «older» than its sister complex. From the fact that all the chromosomes of the one nucleus have the typical appearance of metaphasic intensely staining pycnotic bodies and differ sharply from all the chromosomes of the sister nucleus which display a loose structure and are slightly basophilic, we come to the conclusion that at the anaphase the chromosomes are arranged with respect to the poles of the spindle so as to ensure the passing of all the «young» chromosomes (the pycnotic ones) to the one pole of the spindle, while all the «old» (loose) ones pass to the other. Mitosis is thus a division differential with regard to the state of the chromosomes.

Upon an analysis of the facts communicated above, as well as of the conclusions drawn from them we arrive at a following idea. The sister chromosomes that pass at mitosis to opposite poles are not identical to each other. While strictly corresponding to each other structurally, the two chromatids that arise at the early prophase differ in their properties. An explanation is found in the modern idea of the self-perpetuation of chromosomes ([2–4]).

According to the concept which is based upon the knowledge of the structure of the protein compounds, each ultimate element of the chromosome builds in its neighbourhood from the surrounding karyoplasm another element strictly

corresponding to it; each chromosome element forms autocatalytically from the substances contained in the karyoplasm its «copy», or a «twin» which is identical to it. Daughter chromosomes are thus newly formed «copies» of the initial chromosomes rather than mere halves of the maternal chromosome.

If we accept Koltzoff's hypothesis, the differences between the two chromosomes arisen in mitosis are to be regarded as differences between the initial, «old» chro-

Fig. 2. *Actinophris sol.* Gamete formation. Difference in state of chromosomes in metaphases of sister nuclei ([9], Fig. 151)

mosome which had served as a basis for the autocatalytic process, and its «young» twin, or «copy». The two chromosomes arisen in mitosis are related to each other as mother to daughter rather than as two sisters.

The maternal and the daughter chromosomes, *i. e.* the initial chromosome and the twin formed along with it, while strictly corresponding to each other in structure, differ from each other in their physical-chemical characteristics, and—as it seems to me—their protein component as well. These differences are responsible for the difference in their density, in their bond with thymonucleic acid, and in the mode of spiralization. The total of these differences leads to a difference in the cycles of the initial (maternal) chromosome and its «twin», the daughter chromosome. The maternal chromosome and the daughter chromosome thus form a heterocyclic system.

The high degree of heterocyclicity of the sister nuclei is borne out with a surprising clarity in studies on *Algae*, *Fungi* and *Protista* ([5–10]) (*Stylorhinchus longicolis*, *Cylindrocystis Brebissonii*, *Amoeba diploidea*). The structure and behaviour of the sister nuclei arising from the progamic mitosis in *Actinophris sol.* are shown in Fig. 2. The differentiation of the cycles of the sister nuclei is observed at all stages preceding reduction and fusion. In the sister metaphases—just like those in the cortical parenchyme of potato tubers—the one complement contains loose slightly basophilic chromosomes, while pycnotic, intensely stained chromosome bodies are formed in the other.

Of special importance is the fact that in many *Algae*, *Fungi* and *Protista* the differentiation of the sexual state of the cells takes place in such mitosis.

In his discussion of this fact Hartmann ([8]) wrote as follows: «Dann folgt aber daraus, dass hier durch eine einfache somatische Kernteilung, bei der an dem Kern keine genotypische Aenderung eintritt, die Tochterkerne eine männliche und weibliche Tendenz erhalten... Dieser Fall erscheint theoretisch deshalb besonders wichtig, weil er zeigt, dass die sexuelle Differenzierung, die Herbeiführung einer sexuellen Tendenz, an eine Kernteilung gebunden sei, ohne dass dabei genotypische Verschiedenheiten im Chromosomenbestand zustande kommen» (page 17).

From an analysis of the facts communicated above the following concept is to be arrived at. The differentiation of the cell nuclei involving the sexual state in lower plants and *Protista* proceeds by way of separation of the elements of the system «maternal chromosome—daughter chromosome». The difference in the cyclic state of the chromosomes contained in the sister nuclei, responsible for the differences in metabolism ([13], [14]), cause differences in the sexual state of nuclei and cells.

The source of differentiation of the sexual state of the cell nuclei thus lies in the fundamental property of the nucleus—the capacity of the chromosomes to self-perpetuation. The heterocyclicity of the «maternal-daughter» chromosome components arising in the course of the process of self-perpetuation had been the first step in the evolution of sex.

Differences in the physiological state of the heterocyclic maternal and daughter chromosomes cause, I believe, differences in the structure and behaviour of the primary and secondary nuclei formed in the promitotic division (*Aulacantha scolymantha* [7]), as well as differences in the sister nuclei formed in amitosis in *Amoeba mira* [6]).

I believe also that differences in the structure and behaviour of the vegetative and generative nuclei in the pollen tube in plants depend on the fact that they are composed of heterocyclic maternal and sister chromosomes.

The analysis of the state of the chromosomes in binucleate cells makes us assume that a prolonged functioning of the chromosomes (its «ageing») leads to a reduced capacity of its protein component to bind thymonucleic acid. The chromosomes undergo mitosis when they are undercharged with thymonucleic acid.

Data on the amount of thymonucleic acid in young and old cultures of *Spirillum volutans* [11]) are in good accord with our observations on the «young» and «old» chromosomes in the parenchyme of potato tubers. A study on the influence of age upon heterochromatization [12]) has demonstrated, moreover, the existence of a shift in the state of the chromosomes toward the mitotic end, taking place with the age of the parents.

By comparing these data we arrive at a conclusion that ageing chromosomes pass into a heterocyclic condition which is characterized by a weaker bond with thymonucleic acid.

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REFERENCES

[1] E. Heitz, Planta, **12**, 4, 774 (1931). [2] N. K. Koltzoff, Biolog. Zbl., **48**, 345 (1928). [3] A. A. Shmuck, C. R. Acad. Sci. USSR, XXII, No. 6 (1939). [4] M. Delbrück, Cold Spring Harbor Symposia, **9**, 122 (1941). [5] K. Belar, Biolog. Zbl., **41**, 385 (1921). [6] Idem, Der Formwechsel der Protistenkerne, Jena, 1926. [7] M. Hartmann, Die Konstitution der Protistenkerne, Jena, 1911. [8] Idem, Fortpflanzung und Befruchtung, Berlin, 1929. [9] G. Poljansky, Arch. f. Protistenkunde, **81**, 3, 420 (1934). [10] H. Kniep, Die Sexualität der niederen Pflanzen, Jena, 1928. [11] А. Н. Белозерский, Микробиология, **10**, 2, 185 (1941). [12] А. А. Прокофьева-Бельговская, Журн. общ. биол., **6**, 2, 93 (1945). [13] T. Caspersson, Naturwiss., **29**, 33 (1941). [14] C. D. Darlington, Nature, **149**, 66 (1942).

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PLANT PHYSIOLOGY

EFFECT OF SURFACE-ACTIVE SUBSTANCES UPON INTENSITY OF FIXATION OF ATMOSPHERE NITROGEN BY NITROGEN-FIXING BACTERIA

By M. V. FEDOROV

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In our previous publication ([2]) we have reported data on the influence of the structure of organic substance and of the energy stored in it upon the intensity of the fixation of the atmosphere nitrogen by nitrogen-fixing bacteria. The intensity was found to be proportional to the supply of chemical energy. The only exception to this rule was offered by alcohols where fixation energy proved considerably inferior. This anomalous behaviour of alcohols could be due to nothing else than to their surface activity.

As well known, Traube ([6]) has demonstrated that in the members of the homologous series capillary activity grows in an approximate proportion $1 : 3 : 3^2 : 3^3$. This empirical rule holds true for biological processes as well. Führer was able to show ([7]) that the concentration of alcohols necessary for the narcosis of a frog's heart obeys Traube's rule. In our case, nitrogen fixation intensity with ethyl alcohol was only twice as high as with propyl alcohol; this deviation from Traube's rule may be explained by the possible relation between the activity of *Azotobacter agile* and the surface tension between the protoplasm and the aqueous medium, instead of that measured at the medium—air surface. In order to find out the exact state of affairs we arranged new experiments where inverted sugar was used as the source of energy at a concentration of 0.1 mol. (1.8 g per 100 ml of the medium) and iso-amyl, iso-butyl and iso-propyl alcohols were introduced as surface-active substances. Our modification of the nutrient solution served as a source of mineral medium. It was poured into Ehrlenmeier's flasks of 300 ml capacity, 100 ml per flask, and was sterilized in an autoclave at a pressure of 0.75 atmosphere during half an hour. The alcohols were introduced after sterilization. Inoculation was done by introducing 1 ml of the *Azotobacter* suspension in sterile tap water. The flasks were kept at 25—30° for 30 days. After the end of experiments the residual sugar and total nitrogen were determined in the nutrient solution. The results of the analyses are presented in the table.

The results obtained show that both the intensity of atmosphere nitrogen fixation and respiration intensity drop along with the increase in the concentration of the alcohol in the medium. With a concentration of iso-amyl alcohol making up only 0.005 mol. nitrogen fixation is 95.3 per cent to the control. When the concentration is raised to a tenfold (0.05 mol.), it is only 52.3 per cent. The respiration intensity drops at about the same rate. It follows from these data that both nitrogen fixation and respiration take place in the cell under the participation of active surfaces.

Intensity of Atmosphere Nitrogen Fixation by *Azotobacter* in the Presence of Surface-active Substances

Concentration of alcohol, mols	Amount of sugar spent during experiment, g	Amount of alcohol consumed during experiment, g	Amount of atmosphere nitrogen bound, mg			Efficiency of nitrogen fixation, % to control	Efficiency of respiration, % to control
			pro 1 g of sugar in individual cultures	pro 1 g of sugar on the average	pro 1 g molecule of sugar		
Iso-amyl alcohol							
0.005	1.49	0.0	7.16	7.10	1278.0	95.3	86.4
	1.62	0.0	7.04				
0.010	1.36	0.0	4.73	4.31	775.8	57.8	83.9
	1.66	0.0	3.89				
0.050	0.90	0.0	4.09	3.90	702.0	52.3	51.4
	0.95	0.0	3.71				
0.100	0.64	0.0	0.61	0.72	129.6	9.6	37.9
	0.72	0.0	0.83				
0.200	0.20	0.0	0.28	0.27	48.6	3.6	15.3
	0.55	0.0	0.25				
Iso-butyl alcohol							
0.005	1.80	0.0	7.38	7.37	1326.6	98.9	100.0
	1.80	0.0	7.37				
0.010	1.75	0.0	6.87	6.74	1213.2	90.8	97.5
	1.76	0.0	6.60				
0.050	1.37	0.0	6.64	6.71	1207.8	90.1	74.7
	1.32	0.0	6.78				
0.100	0.98	0.0	4.81	4.69	844.2	62.5	53.1
	0.93	0.0	4.57				
0.200	0.68	0.0	0.14	0.11	19.8	1.5	38.0
	0.68	0.0	0.07				
Iso-propyl alcohol							
0.100	1.75	0.0	7.0	7.20	1296.0	96.7	97.3
	1.75	0.0	7.40				
0.200	1.45	0.0	4.92	4.52	813.6	60.6	80.6
	1.45	0.0	4.11				
0.300	0.90	0.0	2.20	2.07	372.6	27.6	50.0
	0.90	0.0	1.95				
Control (without alcohol)							
0	1.80	0.0	7.14	7.44	1340.2	100.0	—
	1.80	0.0	7.74				

A similar regularity was established for iso-butyl and iso-propyl alcohols. The suppression of nitrogen fixation by 62 per cent for iso-butyl alcohol was found with a concentration of 0.100 mol. (twice that in the case of the iso-amyl alcohol), while in the case of iso-propyl alcohol the corresponding concentration was found to be 0.200 mol. (twice that in the case of iso-butyl alcohol). It follows from these data that when passing from a lower member of the homologous series to a higher one, a twofold reduction of the concentration is necessary to attain a similar effect. The relation does not rigorously obey Traube's rule, but the regular reduction of the concentration to a half is beyond any doubt.

Still more interesting were the results with the further increase in the concentration of alcohols. For example, with an concentration of iso-butyl alcohol amounting to 0.200 mol. the respiration intensity was equal to 38 per cent to the control, while the nitrogen fixation intensity was 1.5 per cent only. Similar results were found for the iso-propyl alcohol. One should conclude from these facts that Blom's hypothesis ([3]) which implies that the catalytic surface of active iron takes part in nitrogen fixation, must be regarded as lacking any foundation. The catalytic apparatus of atmosphere nitrogen fixation and that of respiration are different, inasmuch as they react in a different way to the introduction into solution of surface-active substances.

The difference in the degree of the inhibition of the process of nitrogen fixation by identical concentrations of alcohol may be due to the following two causes: 1) a different degree of absorption by the catalytic surface involved in nitrogen fixation, or 2) a different surface of molecules differing as to their cross-sections. It had been noticed as far back as by Warburg that, despite the great iso-narcotic concentration of the narcotica in solution, the amounts adsorbed by carbon appear to be rather similar in all instances. This observation enabled him to suggest a rather simple explanation of the mechanics of narcosis in heterogeneous catalysis. According to Warburg, the process essentially consists in that the molecules of the narcotic substance, being adsorbed in the form of a solid monomolecular layer on the surface of the catalyst, do correspondingly reduce its free surface.

If alcohols had been equally adsorbed on the catalytic surface involved in nitrogen fixation by *Azotobacter*, the suppression of the process should have been proportional to the cross-section areas of their molecules plus interspaces (calculated after Kopp's formula) and have shown the relation $0.322 : 0.385 : 0.453 = 1 : 1.2 : 1.40$. The analyses have shown, however, that equal suppression of the process of nitrogen fixation is attained with relative concentration $2^2 : 2 : 1$. Thus, the catalytic surface of the cell taking part in nitrogen fixation adsorbs these alcohols to a different degree.

If we are to take into consideration the different areas of the molecules of these substances, we can readily find the actual degree of adsorption of these alcohols by the catalytic surface taking part in the nitrogen fixation. It must be proportional to the following values:

(0.322×4) :	(0.385×2) :	$0.453 = 1.0 : 0.590 : 0.352$
iso-propyl alcohol	iso-butyl alcohol	iso-amyl alcohol

From these figures it is easy to see that the degree of the adsorption of these alcohols is nearly proportional to their capillary activities.

From the relative adsorption of these substances and from the degree of suppression of the fixation of atmosphere nitrogen, it is possible to make an estimate of the catalytic surface involved in nitrogen fixation. Indeed, when the concentration of iso-amyl alcohol is increased from 0.05 to 0.1 mol., nitrogen fixation drops from 52.3 to 9.6 per cent. With a similar increase in the concentration of iso-butyl alcohol, nitrogen fixation drops from 90.7 to 62.5 per cent. Identical doses of iso-butyl and iso-amyl alcohol resulted in a difference in suppression amounting to 15.5 per cent. If we assume that the molecules of the 0.05 mol. of iso-butyl alcohol form a monomolecular layer over the surface of the catalyst, blocking a

certain fraction of the latter, then in view of a lower adsorption of this substance as compared with that of iso-amyl alcohol (1.70 times) we are to assume that the molecules of the latter will suppress the process to an equal degree only in the case where its concentration is 1.70 times lower. The areas covered by the monomolecular layer of these substances should be: for iso-amyl alcohol $0.453\ m\mu^2 \cdot 0.303 \cdot 10^{23} = 0.1375 \cdot 10^{23}\ m\mu^2$, and for iso-butyl alcohol, $0.385\ m\mu^2 \cdot 0.303 \cdot 10^{23} = 0.1166 \cdot 10^{23}\ m\mu^2$. Owing to a greater adsorption of iso-amyl alcohol, the first value should be reduced 1.70 times. With this correction the area occupied by the molecules of iso-amyl alcohol, apt to produce an effect similar to that produced by 0.05 mol. of iso-butyl alcohol, should be equal to $0.1375 \cdot 10^{23}\ m\mu^2 : 1.70 = 0.0809 \cdot 10^{23}\ m\mu^2$. It follows from this that the extra-surface covered by iso-amyl alcohol as compared to iso-butyl alcohol, is equal to $0.1375 \cdot 10^{22}\ m\mu^2 - 0.0809 \cdot 10^{23}\ m\mu^2 = 0.0566 \cdot 10^{23}\ m\mu^2$. This excess of the blocked surface is responsible for the nitrogen fixation being reduced by 15.5 per cent in the experiment with amyl alcohol. Thus, the entire surface of the catalyst which was involved in nitrogen fixation in the control experiment should answer the following relation:

$$\frac{0.0560 \cdot 10^{23}\ m\mu^2 \cdot 100}{15.5} = 0.360 \cdot 10^{23}\ m\mu^2, \text{ or } 3.6 \cdot 10^4\ m^2$$

This catalytic surface could suffice for the fixation of 14 mg of atmosphere nitrogen. Accordingly, in order to bind one mg of atmosphere nitrogen, *Azotobacter* probably develops a catalytic surface of about 2600—3000 m^2.

If we are to assume the distances between the centres of the molecules of nitrogen to be equal to 0.122 mμ, their cross-section areas should be about 0.047 $m\mu^2$. An area of 2600 m^2 will give place to about $6.0 \cdot 10^{20}$ molecules of nitrogen, and in 1 mg of the nitrogen assimilated there will be contained $0.22 \cdot 10^{20}$ molecules only. Rather curious figures are thus obtained. The surface of the catalyst of nitrogen fixation appears to be greater than that covered by a monomolecular layer of nitrogen subject to assimilation.

We ascribe to these calculations no absolute value, but we are nevertheless obliged to conclude that activation of the nitrogen molecules is probably attained with greatest difficulty, and that only a certain fraction of these enter reaction with other elements to form products of nitrogen fixation. It is quite natural to suppose, therefore, that such a reproduction outside the cell of a process, connected with a tremendous catalytic or rather structural area of the cell, involves great difficulties. This assumption is in accord with the facts found in literature. Burk ([4]), Roberg ([5]) and others were unable to produce any appreciable amount of fixed nitrogen with the aid of cell sap of *Azotobacter*.

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Received
3.VII.1945.

REFERENCES

[1] A. N. Bach, Ermolieva and Stepanian, C. R. Acad. Sci. URSS, I, No. 1 (1934). [2] М. В. Федоров, Микробиология, XIV, в. 2 (1945). [3] J. Blom, Biochem. Z., **34**, 385 (1928). [4] D. Burk, Биохимия, **2**, 312 (1937). [5] M. Roberg, Jahrb. f. wissensch. Bot., **83**, 567 (1936). [6] J. Traube, Liebigs Annal., **265**, 27 (1891). [7] H. Führner, Bioch. Z., **120**, 143 (1921).

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EXPERIMENTAL MORPHOLOGY

LIMB REGENERATION IN ADULT FROG

By L. V. POLEZHAJEV

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Regeneration ability of limbs in tadpoles vanishes at a definite stage of metamorphosis. A complete regeneration of typical limbs at later stages of metamorphosis may be obtained, however, by applying various experimental methods ([2], [3]). In order to restore the regeneration ability it is necessary to induce the regeneration blastema which arises in the case of dedifferentiation of the mesodermal tissues of the organ's remain. Dedifferentiation takes place if the destruction of the tissues of the amputation wound surface is stimulated ([4], [6], [7]) by means of various experimental procedures.

Rose ([8],[9]) succeeded in restoring the regeneration ability of limbs of adult frogs. In order to prolong the wound stimulation he treated the amputation surfaces of the limbs with saturated sodium chloride solution. By experiments on tadpoles I was brought to the conclusion that the method of salt treatment, and a number of old and new methods of treatment (traumatization, transplantation) give a similar result, stimulating the dedifferentiation of the mesodermal tissues of the amputation wound surface, thus leading to the formation of a blastema ([5]). My idea was that for obtaining regeneration of limbs in adult frogs it was necessary to apply such methods as would cause destruction and dedifferentiation of differentiated tissues, and formation of the blastema. In order to verify this assumption, a number of experiments were carried out in spring and summer 1944. The purpose of these experiments was to induce regeneration of limbs of adult frogs. The results obtained are described below.

Experiments on *Rana temporaria*. Frogs at their second year after metamorphosis were taken for the experiments. Their body length, as measured from the anterior end of the head to the posterior end of urostyle, was 30—45 mm. In all the experiments were amputated the right forelimbs in the middle or distal part of the forearm, *i. e.* between the hand and the ulnar junction.

1. The amputation surfaces of the experimental animals were treated with saturated sodium chloride solution after Rose's method. Unfortunately, the result of this set of experiments escaped evaluation, since all the animals perished accidentally before the end of the experiment.

2. During 22 days the amputation surfaces were daily traumatized with a needle after which they were treated with saline during 3—7 minutes. In all 8 cases regeneration set in. The regenerates developed were abnormal in appearance, in some cases bearing lateral outgrowths and tubercles (Fig. 1, *b*). They were much unlike a typical forelimb of a frog (Fig. 1, *a*).

3. During 7 days the amputation surfaces were daily treated with 10 per cent iodine alcohol solution. This mode of treatment soon resulted in the death of the experimental animals, while in those which survived the wounds remained open

during 2—3 months. Nevertheless, 7—11 months after amputation regeneration of abnormal limbs was recorded (Fig. 1, *c*).

4. The skin and muscles of a frog were conserved after Filatov's method (1) during 7 days at a temperature of 2—4° C. The fragments of preserved skin were then transplanted on to the back (extensor) surface of the forearm. After the healing

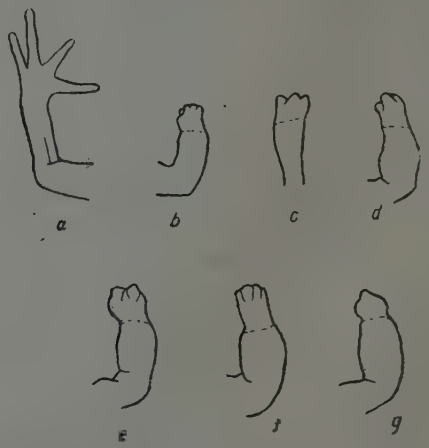

Fig. 1. Regenerated part of limb dotted

of transplanted pieces, the legs were amputated within the transplants. 2 months after the operation in all the 12 animals that survived regeneration cones developed. Unfortunately, some of the frogs lost their regeneration cones because of the mechanical pressure of the legs on the bottom and walls of the aquarium. However, from 8 to 12 months after the amputation abnormal limbs regenerated in many frogs (Fig. 1,*d*), which were not differentiated at their distal ends.

5. Preserved muscles taken from an extremity of another frog were grafted between the skeleton and the muscles of the forearm. In a few days after the operation the limbs were amputated in such a way that part of the tissues transplanted was inserted into the wound surface. In 9 frogs out of 10 abnormal limbs regenerated 8—12 months after the amputation. In the most successful cases the regenerated parts had the shape of a spatula with rudimentary fingers (Fig. 1,*e*).

6. In order to induce intense decay of the tissues before the amputation, a tight ligature was put on the distal part of the forearm. The ligature caused a strong inflammation, hyperaemia and swelling of the legs. After 10—14 days the hands were found to be necrotized, and the legs were amputated at the line of the ligature. 1.5 months after the amputation in all the 15 cases under experiment regeneration cones had developed. The latter gave rise to abnormal extremities that developed 7—11 months after operation (Fig. 1, *f*). They were rather well-shaped as compared to those in the other experimental sets.

7. A ligature was applied to the leg. After amputation the wound surface was during 19-day-period daily treated for 3—7 minutes with a salt solution. Regeneration buds were formed in all 3 cases studied. The size and developmental features of these regenerates did not differ from those of the preceding experiment with ligature.

8. The control consisted in an amputation of the limb alone. 3 weeks after amputation 8 animals were alive and all of them developed regeneration cones 2 months after operation. Several frogs died during the last two months and all of them had regeneration buds. After 12 months one frog developed an abnormal extremity (Fig. 1, *g*).

These experiments have shown that in *Rana temporaria* on the second year after metamorphosis the limbs are able to regenerate when amputation is practised in the region of the forearm. Regeneration takes place not only under experimental conditions when various treatments are applied, but also in the case of a simple amputation. Under experimental conditions a certain stimulation of regeneration is observed, which leads to the development of more differentiated extremities. In all the experimental sets only abnormal limbs were regenerated, always with deficient distal parts and mostly without fingers. Only in the most successful cases rudiments of fingers developed that were fused with one another

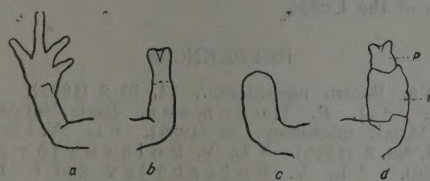

Fig. 2. *p*, regenerate; *T*, skin transplanted

(Fig. 1, *f*). The best results were obtained in experiments with ligature and grafting of preserved muscle fragments.

The fact of regeneration of limbs in the control deserves special attention. It shows that the regeneration ability which is lost by the frog in the process of metamorphosis is spontaneously restored in a new form. This fact makes us revise our old conception as to the mechanism underlying the loss of the regeneration ability of the limbs in *Anura*. Apparently this loss should be connected not only with the degree, but rather with the intensity of histological differentiation which prevents the dedifferentiation of tissues and the formation of a blastema. Histological differentiation of the limbs during metamorphosis proceeds more intensely than after metamorphosis is over. The possibility of tissue dedifferentiation during metamorphosis must therefore be lower than upon metamorphosis. The growth of the extremity tissues may also have a favourable effect upon dedifferentiation of the tissues.

The limbs regenerated in tadpoles are typical, while those developed in frogs are abnormal ones. This difference may be due to the changes in the morphogenetic abilities of the leg tissues, as well as to changes in the internal humoral environments of the animal, as it passes from the larval stage to the adult stage.

Experiments on *Bombina bombina*. The right or the left forelimbs were amputated between the hand and the ulnar joint. Little animals at their second year of life after metamorphosis were used for the experiments, with a body length of 20—24 mm, and adult sex-mature animals, 40—42 mm in length.

In the case simple amputations were practised on young animals, the extremities regenerated in all cases; they were abnormal in appearance (Fig. 2,*a*), being either outgrowths of indefinite shape, or extremities with a spatula-like foot, or monodidactylous (Fig. 2, *b*). In adult animals of this species a mere amputation did not lead to the regeneration of limbs. The wounds were smoothly healed. Very small tubercles developed in rare cases only (Fig. 2, *c*). When preserved skin was grafted on to the forearm in the form of wristband with a subsequent amputation in the region of the transplant, the stimulation of the formation of a blastema and the regeneration of an extremity were observed in the young animals. In the adults the regeneration of outgrowths or abnormal extremities was observed under similar experimental conditions (Fig. 2, *d*).

Thus, the regeneration ability in *Bombina* is partly preserved after metamorphosis, to disappear completely in the course of time. The experiments show that under definite experimental conditions it may be restored, to a certain degree at least.

The data reported above confirm Rose's ([8], [9]) statement to the effect that the limbs in adult frogs are able to regenerate. The fact that a mere amputation, with-

out any additional treatment, leads to the regeneration of extremities in *Rana temporaria* (our data), whereas under similar experimental conditions the legs of *Rana clamitans* and *Rana catesbiana* fail to regenerate (Rose's data), should be put down to the specific differences of the experimental material.

Experiments on *Bombina* carried out with the aid of a different method went to confirm the data of Rose who showed that the lost regeneration ability in the adult *Anura* may be restored under experimental conditions.

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Received
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REFERENCES

[1] В. П. Филатов, Вестн. офтальмол., 11, № 2 (1937). [2] L. D. Liosner, Roux' Arch., 124 (1931). [3] Л. В. Полежаев, Биол. журн., 2 (1933). [4] L. W. Polé jaiev, Arch. d'Anat. microscop., 32 (1936). [5] L. V. Polezhajev, C. R. Acad. Sci. URSS, XLVIII, No. 3 (1945). [6] L. V. Polezhajev and G. I. Ginzburg, ibid., XXIII, No. 7 (1939). [7] L. V. Polezhajev and I. I. Morosov, ibid., XXX, No. 7 (1941). [8] S. M. Rose, Proc. Soc. exp. Biol. and Med., 49, 408 (1942). [9] S. M. Rose, J. exp. Zool., 95 (1944).